Obstruction theory for parameterized higher WZW terms

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Motivation

Classical fact: Obstruction to globalizing a form $\omega \in \Omega^{p+2}_{\mathrm{cl}}(\mathbb{R}^n)$ over an n-manifold is existence of $\mathrm{Stab}_{\mathrm{GL}(n)}(\omega)$-structure.
Example: $\omega \in \Omega^3(\mathbb{R}^7)$ the associative 3-form $\Rightarrow$ G_2-structure.

Questions: What happens as...
1.) ...forms are prequantized to Deligne cocycles $\mathbf{L}$?
2.) ...base is allowed to be a higher étale stack?
A) What are the obstructions to existence of a globalization?
B) What is group stack of symmetries of any given globalization?

Application:
All Green-Schwarz-type super p-brane sigma-models are controlled by $\mathbf{L}^{p+2}_{\mathrm{WZW}}$ globalized over a super-spacetime;
for the D-branes and for the M5-brane the base is a higher stack modeled on the homotopy fiber of $\mathbf{L}^{\mathrm{F1}}_{\mathrm{WZW}}$, $\mathbf{L}^{\mathrm{M2}}_{\mathrm{WZW}}$, respectively.
A) Obstruction to global existence:
classical anomalies (was completely open)
B) Symmetries of given globalization:
BPS charge extended superisometries (was only known rationally)

Blueprint: ordinary geometric prequantization

Consider $\omega := dp_i \wedge dq^i \in \Omega^2_{\mathrm{cl}}(\mathbb{R}^{2n})$ and $\mathbf{L} := p_i \wedge dq^i$, then:

1. definite globalization (X, ω^X):	alm. symplectic structure;
2. definite globalization $(X; \mathbf{L}^X)$:	prequantum line bundle;
3. symmetry group of (X, ω^X):	symplectomorphism group;
4. symmetry group of $(X, \mathbf{L}^X)$:	quantomorphism group.

Since $\mathbf{L}$ has automorphisms, where ω does not, quantomorphisms form (central) extension:

$$\bigoplus_{\pi_0(X)} (\mathbb{R}/\Gamma) \hookrightarrow \mathrm{QuantMorph}(X, \mathbf{L}^X) \longrightarrow\!\!\!\!\!\rightarrow \mathrm{HamSympl}(X, \omega^X)$$

On the Lie algebra level this is the Poisson bracket extension:

$$0 \to H^0(X) \longrightarrow \mathfrak{pois}(X, \omega^X) \longrightarrow \mathrm{HamVect}(X, \omega^X) \to 0\,.$$

For (X, ω^X) a symplectic vector space, this is the Heisenberg extension.

Higher differential geometry

Lift classical theory from the category $\mathrm{SmoothMfd}$ to the homotopy theory $\mathbf{H}$ of simplicial sheaves over smooth manifolds ("smooth ∞-groupoids", "higher smooth stacks").

Theorem (classical+Lurie'12): A_∞-group stacks are equivalently loopings of pointed connected higher stacks:

$$\mathrm{Grp}(\mathbf{H}) \underset{\mathbf{B}}{\overset{\Omega}{\underset{\longrightarrow}{\overset{\longleftarrow}{\simeq}}}} \mathbf{H}^{*/}_{\geq 1}$$

Theorem [dcct]: $\mathbf{H}$ is *cohesive*, the derived global section coreflection $\flat := \mathrm{Lconst} \circ \Gamma$ produces moduli stacks of flat G-principal connections for any A_∞-group stack G.
The double homotopy fiber of the $\flat$-counit is the higher

Maurer-Cartan form:

$$\begin{tikzcd} G \arrow[r, "\theta_G"] & \flat_{\mathrm{dR}}\mathbf{B}G \arrow[d] & \\ & \flat\mathbf{B}G \arrow[r] & \mathbf{B}G \end{tikzcd}$$

Higher prequantization

The Dold-Kan correspondence $\mathrm{DK} : \mathrm{Ch}_{\bullet \geq 0} \stackrel{\simeq}{\to} \mathrm{Ab}^{\Delta^{\mathrm{op}}} \to \mathrm{Set}^{\Delta^{\mathrm{op}}}$ includes traditional sheaf hypercohomology into $\mathbf{H}$.

Write:
$$\begin{aligned} \mathbf{B}^{p+1}(\mathbb{R}/\Gamma) &:= \mathrm{DK}((\underline{\mathbb{R}}/\Gamma)[p+1]); \\ \flat_{\mathrm{dR}} \mathbf{B}^{p+2}\mathbb{R} &:= \mathrm{DK}(\mathbf{\Omega}^1 \stackrel{d}{\to} \cdots \stackrel{d}{\to} \mathbf{\Omega}^{p+2}_{\mathrm{cl}}) \end{aligned}$$

Proposition [dcct]: Homotopy pullback of MC-form $\theta_{\mathbf{B}^{p+1}(\mathbb{R}/\Gamma)}$ along the global differential form inclusion

$$\begin{tikzcd} & \mathbf{B}^{p+1}(\mathbb{R}/\Gamma)_{\mathrm{conn}} \arrow[r] \arrow[d, "F_{(-)}"] & \mathbf{B}^{p+1}(\mathbb{R}/\Gamma) \arrow[d, "\theta_{\mathbf{B}^{p+1}(\mathbb{R}/\Gamma)}"] \\ X \arrow[ur, dashed, "\mathbf{L}"] \arrow[r, "\omega"] & \mathbf{\Omega}^{p+2}_{\mathrm{cl}} \arrow[r] & \flat_{\mathrm{dR}} \mathbf{B}^{p+2}\mathbb{R} \end{tikzcd}$$

is given by the Deligne complex:

$$\mathbf{B}^{p+1}(\mathbb{R}/\Gamma)_{\mathrm{conn}} \simeq \mathrm{DK}[\Gamma \hookrightarrow \mathbf{\Omega}^0 \stackrel{d}{\to} \mathbf{\Omega}^1 \stackrel{d}{\to} \cdots \stackrel{d}{\to} \mathbf{\Omega}^{p+1}]$$

Fact: $X \stackrel{\mathbf{L}}{\to} \mathbf{B}^{p+1}(\mathbb{R}/\Gamma)_{\mathrm{conn}}$ has holonomy over closed mfd. Σ:

$$\begin{tikzcd} {[\Sigma, X]} \arrow[r, "{[\Sigma,\mathbf{L}]}"] \arrow[rr, bend left, "\text{WZW action functional}"] & {[\Sigma, \mathbf{B}^{p+1}(\mathbb{R}/\Gamma)_{\mathrm{conn}}]} \arrow[r, "\text{fiber integration}"'] & \mathbf{B}^{p+1-\dim(\Sigma)}(\mathbb{R}/\Gamma)_{\mathrm{conn}} \end{tikzcd}$$

Infinitesimal symmetries

Theorem [FRS13b]: For $X \in \mathrm{SmthMfd}$, infinitesimal symmetries of $(X, \mathbf{L})$ form L_∞-algebra extension of vector fields by the abelian L_∞-algebra on the de Rham complex, classified by the L_∞-cocycle given by $\iota_{(-)}\omega$: there is a homotopy fiber sequence

$$\begin{tikzcd} \mathbf{\Omega}^\bullet[p] \arrow[r] & \mathfrak{poiss}(X, \mathbf{L}) \arrow[d] & \\ & \mathrm{HamVect}(X, \omega) \arrow[r, "\iota_{(-)}\omega"] & \mathbf{\Omega}^\bullet[p+1] \end{tikzcd}$$

Corollary. Under 0-truncation τ_0 (chain homology) this becomes a central extension of Lie algebras

$$0 \longrightarrow H^p_{\mathrm{dR}}(X) \longrightarrow \tau_0\mathfrak{poiss}(X, \mathbf{L}) \longrightarrow \mathrm{HamVect}(X, \omega) \to 0\,.$$

Regarding $\mathbf{L}$ as a WZW term, then $\tau_0\mathfrak{poiss}(X, \mathbf{L})$ is the Dickey bracket on Noether currents[1] for target space symmetries of the sigma-model with WZW term $\mathbf{L}$ (compare AGIT'89).

[1] With Igor Khavkine.

Finite symmetries

Definition:

1.) $\mathrm{conc} : [X, \mathbf{B}^{p+1}(\mathbb{R}/\Gamma)_{\mathrm{conn}}] \longrightarrow (\mathbf{B}^p(\mathbb{R}/\Gamma))\mathbf{Conn}(X)$
projection on moduli of vertical differential forms;

2.) $\mathbf{QuantMorph}(X, \mathbf{L}) := \mathbf{Stab}_{\mathbf{Aut}(X)}(\mathrm{conc}(\mathbf{L}))$
homotopy stabilizer group;

3.) $\mathbf{HamSympl}(X, \mathbf{L}) := \mathrm{im}_1(\mathbf{QuantMorph}(X, \mathbf{L}) \to \mathbf{Aut}(X))$
1-image of quantomorphisms in automorphisms;

4.) $\mathbf{Heis}_G(X, \mathbf{L}) := \rho^*\mathbf{QuantMorph}(X, \mathbf{L})$
its pullback along any G-action $\rho : G \to \mathbf{Aut}(X)$.

Theorem [FRS13a]: There is a homotopy fiber sequence:

$$\begin{tikzcd}
(\mathbf{B}^{p-1}(\mathbb{R}/\Gamma))\mathbf{FlatConn}(X) \arrow[r] & \mathbf{QuantMorph}(X, \mathbf{L}) \arrow[d, two heads] & \\
& \mathbf{HamSympl}(X, \mathbf{L}) \arrow[r, "\mathbf{KS}"] & \mathbf{B}((\mathbf{B}^{p-1}(\mathbb{R}/\Gamma))\mathbf{FlatConn}(X))
\end{tikzcd}$$

Example: For $p = 0$ this reduces to the traditional Heisenberg-Kostant-Souriau quantomorphism group extension.

Obstruction theory, part I

Theorem [dcct]:
Let G be an A_∞-group stack
and $\mathbf{L} : G \longrightarrow \mathbf{B}^{p+1}(\mathbb{R}/\Gamma)_{\mathrm{conn}}$
with $G \hookrightarrow \mathbf{HamSympl}(X, \mathbf{L})$.
Then the obstruction to a definite parameterization of $\mathbf{L}$ over the fibers of a G-principal ∞-bundle $P \to X$ [NSS12a] is equivalently:
1.) a lift of the structure group through $\mathbf{Heis}_G(X, \mathbf{L}) \to G$;
2.) a trivialization of $\mathbf{KS}(P)$.

Example [FRS13a]: For $G = \mathrm{Spin}$ and $\mathbf{L}^{\langle -,[-,-]\rangle}_{\mathrm{WZW}}$ the traditional WZW term, then

$$\mathbf{Heis}_{\mathrm{Spin}}(\mathrm{Spin}, \mathbf{L}^{\langle -,[-,-]\rangle}_{\mathrm{WZW}}) \simeq \mathrm{String}$$

is the smooth String-2-group, and hence a definite parameterization here is precisely a smooth String-structure.
This gives the geometric interpretation of the cancellation of the Green-Schwarz anomaly due to Distler-Sharpe'07 .

Higher WZW terms

For $\mathfrak{g}$ an L_∞-algebra and $\mu_{p+2} \in \mathrm{CE}^{p+2}(\mathfrak{g})$ an L_∞-cocycle, write $\mathbf{B}G \in \mathbf{H}$ for the $(p+2)$-coskeleton of the simplicial sheaf of flat $\mathfrak{g}$-valued forms on simplices, parameterized by manifolds U:

$$\mathbf{B}G : (U \in \mathrm{SmoothMfd}) \mapsto \mathrm{cosk}_{p+2}(\Omega_{\substack{\mathrm{flat} \\ \mathrm{vert}}}(U \times \Delta^\bullet_{\mathrm{smth}}, \mathfrak{g}))$$

Theorem [FSS10]: μ Lie integrates to $\mathbf{c} : \mathbf{B}G \longrightarrow \mathbf{B}^{p+2}(\mathbb{R}/\Gamma)$

Definition. Denote by

$$\begin{tikzcd} \tilde{G} \arrow[r] \arrow[d, "\theta_{\tilde{G}}"'] \arrow[dr, phantom, "\text{(pb)}"] & G \arrow[d, "\theta_G"] \\ \Omega_{\mathrm{flat}}(-, \mathfrak{g}) \arrow[r] & \flat_{\mathrm{dR}}\mathbf{B}G \end{tikzcd}$$

the homotopy pullback of the MC form on G to globally defined forms.

Theorem [dcct]: $\Omega\mathbf{c}$ underlies a unique prequantization

$$\mathbf{L}^{\mu}_{\mathrm{WZW}} : \tilde{G} \longrightarrow \mathbf{B}^{p+1}(\mathbb{R}/\Gamma)_{\mathrm{conn}}$$

of $\mu(\theta_{\tilde{G}}) \in \Omega^{p+2}(\tilde{G})$. $\boxed{\text{This is the higher WZW term of } \mu.}$

Higher étale stacks (higher orbifolds)

Let now **H** be simplicial sheaves over *formal* manifolds.

Theorem [dcct]: This is *differentially cohesive*, reduction $\Re$ of infinitesimals has a right adjoint $\Im$ ("de Rham stack functor").

Definition. A morphism is infinitesimally étale $\xrightarrow{\mathrm{et}}$ if its $\Im$-unit is a homotopy pullback square.

Definition. For V an A_∞-group stack, a *V-étale stack* is an $X \in \mathbf{H}$ such that there exists a V-cover: $V \xleftarrow{\mathrm{et}} U \xrightarrow{\mathrm{et}} X$.

Definition. The *infinitesimal disk bundle* is:

$$\begin{tikzcd} T_{\mathrm{inf}} X \arrow[r, "\mathrm{ev}"] \arrow[d, "p"'] \arrow[dr, phantom, "\text{(pb)}"] & X \arrow[d] \\ X \arrow[r] & \Im X \end{tikzcd}$$

Theorem [dcct]: 1.) The infinitesimal disk bundle of V trivializes via left translation, with typical fiber the infinitesimal disk $\mathbb{D}^V_e$;
2.) the infinitesimal disk bundle of any V-étale stack is locally trivial and associated to a $\mathrm{GL}(V) := \mathbf{Aut}(\mathbb{D}^V_e)$-principal ∞-bundle: the *frame bundle* $\mathrm{Fr}(X) \to X$.

Obstruction theory, part II

Definition: Given $\mathbf{L} : V \longrightarrow \mathbf{B}^{p+1}(\mathbb{R}/\Gamma)_{\mathrm{conn}}$, a *definite globalization* over a V-étale stack X is $\mathbf{L}^X : X \to \mathbf{B}^{p+1}(\mathbb{R}/\Gamma)_{\mathrm{conn}}$ such that its restriction to infinitesimal disks along $T_{\mathrm{inf}}X \stackrel{\mathrm{ev}}{\to} X$ is a parameterization (as above) definite on $\mathbf{L}|_{\mathbb{D}^V_e}$.

Corollary [dcct]: An obstruction to definite globalization is $\mathbf{Heis}_{\mathrm{GL}(V)}(\mathbb{D}^V_e, \mathbf{L}|_{\mathbb{D}^V_e})$-structure, hence trivialization of the **KS**-class of the frame bundle.

Example: For X an $\mathbb{R}^{2n}$-manifold and $\mathbf{L} = p^i \wedge dq^i$, and for second order infinitesimals, then $\mathbf{Heis}_{\mathrm{GL}(V)}(\mathbf{L}|_{\mathbb{D}^V_e}) \simeq \mathrm{Mp}^c(2n)$ is the Metaplectic^c group.

Example [dcct][2]: For V super-Minkowski spacetime and $\mathbf{L}$ the κ-WZW term for the GS super-string, then $\mathbf{Heis}_{\mathrm{Aut}_{\mathrm{grp}}(\mathbb{D}^V_e)}(\mathbf{L}|_{\mathbb{D}^V_e})$ is a $\mathbf{B}(\mathbb{R}/\Gamma)$-extension of the Lorentzian Spin group $\mathrm{Spin}(d-1,1)$.

[2]With John Huerta.

Higher supergeometry

Let now **H** be simplicial sheaves over formal *supermanifolds*.

Theorem [dcct][3]:
$\exists$ Progression of adjoint (co-)localizations: $\to$
satisfying $\overset{\rightsquigarrow}{\Im} \simeq \Im$.

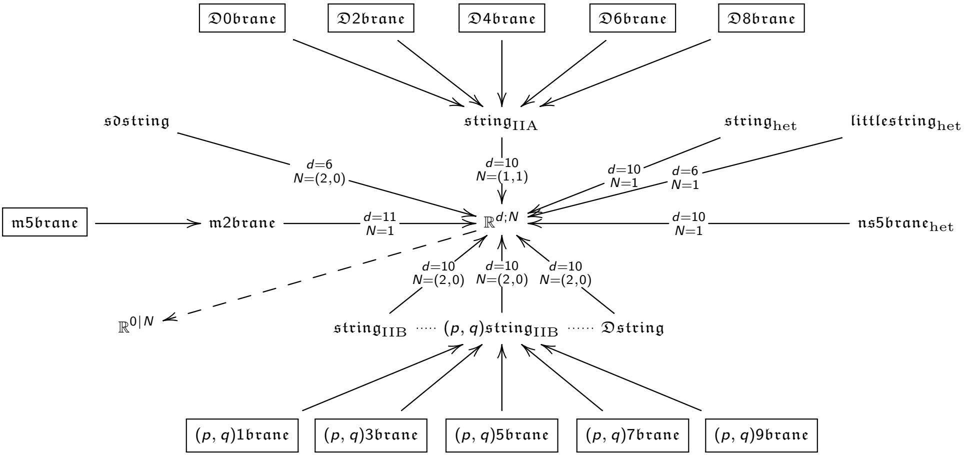
Proposition [dcct]:
For
X a V-étale stack,
then
$\overset{\rightsquigarrow}{X}$ is $\overset{\rightsquigarrow}{V}$-étale stack.

$$
\begin{array}{lccc}
 & \mathrm{id} & \dashv & \mathrm{id} \\
 & \vee & & \vee \\
\text{even} & \rightrightarrows & \dashv & \rightsquigarrow \\
 & \perp & & \perp \\
\text{bosonic} & \rightsquigarrow & \dashv & \mathrm{loc}_{\mathbb{R}^{0|1}} \\
 & \vee & & \vee \\
\text{reduced} & \Re & \dashv & \Im \\
 & \perp & & \perp \\
\text{étale} & \Im & \dashv & \& \\
 & \vee & & \vee \\
\text{shape} & \mathrm{loc}_{\mathbb{R}} & \dashv & \flat \\
 & \perp & & \perp \\
\text{flat} & \flat & \dashv & \sharp \\
 & \vee & & \vee \\
 & \emptyset & \dashv & *
\end{array}
$$

[3] With Dave Carchedi.

Application: GS-WZW terms for super p-branes

Fact Azcárraga-Townsend'89+[FSS13]: The iterative super-L_∞ $b^p\mathbb{R}$-extensions of the superpoint come from the WZW cocycles μ of all the Green-Schwarz-type super-p-branes sigma-models:



M5-brane on M2-brane extended superspacetime I

Regard super-Minkowski spacetime $\mathbb{R}^{d-1,1|N}$ as a super Lie algebra. Write
$$\mu_{p+2} := \overline{\psi}\Gamma^{a_1\cdots a_p} \wedge \psi \wedge e_{a_1} \wedge \cdots e_{a_p} \ \in \mathrm{CE}(\mathbb{R}^{d-1,1|N})\,.$$

Proposition D'Auria-Fré 89: The elements $\mu_4, \mu_7 \in \mathrm{CE}(\mathbb{R}^{10,1|\mathbf{32}})$ satisfy
$$d\mu_4 = 0\ ,\quad d\mu_7 = \mu_4 \wedge \mu_4.$$

Proposition [FSS13]: The M2-brane extended super Minkowski spacetime with $\mathrm{CE}(\hat{\mathbb{R}}^{10,1|\mathbf{32}}) := \mathrm{CE}((\mathbb{R}^{10,1|\mathbf{32}}) \otimes \langle h_3 \rangle, dh_3 = -\mu_4)$ is the L_∞-homotopy fiber of μ_4 and we have

$$\begin{tikzcd}
\hat{\mathbb{R}}^{10,1|\mathbf{32}} \arrow[r, "h_3\wedge\mu_4+\frac{1}{15}\mu_7"] \arrow[d] & b^6\mathbb{R} \\
\mathbb{R}^{10,1|\mathbf{32}} \arrow[r, "\mu_4"] & b^3\mathbb{R}
\end{tikzcd}$$

matching the proposal in BLNPST'97.

M5-brane on M2-brane extended superspacetime II

Theorem [dcct]: For a definite globalization of consecutive WZW terms such as

$$\begin{tikzcd} \widetilde{\hat{X}} \arrow[r, "\mathbf{L}_{\mathrm{WZW}}^{\mathrm{M5}}"] \arrow[d] & \mathbf{B}^6 U(1)_{\mathrm{conn}} \\ X \arrow[r, "\mathbf{L}_{\mathrm{WZW}}^{\mathrm{M2}}"] & \mathbf{B}^3 U(1)_{\mathrm{conn}} \end{tikzcd}$$

$\widetilde{\hat{X}}$ is a $\mathbf{B}^2(\mathbb{R}/\Gamma)_{\mathrm{conn}}$-bundle over X.

Hence a map $\Sigma \to \widetilde{\hat{X}}$ is a pair consisting of

1. a sigma-model field $\phi : \Sigma \to X$;
2. a ϕ-twisted degree-3 Deligne cocycle (twisted 2-gerbe with connection) on Σ.

This is the "tensor multiplet" field content of the M5-brane globalized to a twisted 2-gerbe connection.

M5-brane on M2-brane extended superspacetime III

Proposition above results+ Candiello-Lechner'93: First order integrable definite globalization of $\mathbf{L}_{\mathrm{WZW}}^{\mathrm{M2}}$ implies super-Lorentzian structure with vanishing supertorsion, this in turn implies the vaccuum equations of motion of 11d Einstein gravity., enhances them by cancelling the obstruction to making the M2-brane and M5-brane WZW terms be globally defined.

Proposition [dcct]:
The isometry action on X lifts to an ∞-action on $\widetilde{\hat{X}}$.
Corollary: The infinitesimal symmetries of $\mathbf{L}_{\mathrm{WZW}}^{\mathrm{M5}}$ are an extension of superisometries of spacetime by H^5 of the $K(\mathbb{Z},3)$-bundle underlying $\mathbf{L}_{\mathrm{WZW}}^{\mathrm{M2}}$. Running the Serre spectral sequence, rationally this is $H^2(X) \oplus H^5(X)$.

This is the traditional result for the *M-theory super Lie algebra*, the extension of the superisometries by BPS charges for the M2-brane and the M5-brane Sorokin-Townsend'97. The above analysis gives the finite global symmetries involving various (torsion) corrections to this.

M5-brane on M2-brane extended superspacetime - Outlook

Proposition: M5-cocycle descends equivariantly down to super-Minkowski spacetime

$$
\begin{tikzcd}
 & \vdots \arrow[d, shift left] \arrow[d, shift right] & & \vdots \arrow[d, shift left] \arrow[d, shift right] \\
(dh_3 = -\mu_4) & \hat{\mathbb{R}}^{10,1|\mathbf{32}} \arrow[rr, "h_3 \wedge g_4 + \frac{1}{15}\mu_7"] \arrow[d] & & b^6\mathbb{R} \arrow[d] \\
(dh_3 = g_4 - \mu_4) & \mathbb{R}^{10,1|\mathbf{32}} \arrow[rr, "h_3 \wedge (g_4 + \mu_4) + \frac{1}{15}\mu_7"] \arrow[dr, "\mu_4"'] & & b^6\mathbb{R}/b^2\mathbb{R} \arrow[dl, "p_\rho"] \\
 & & b^3\mathbb{R} &
\end{tikzcd}
$$

By [NSS12a] this rational 4-sphere valued cocycle is in degree-7 twisted cohomology, the twist being the degree-4 class of the supergravity C-field.
This structure of the M-theory C-field was conjectured in Sati'13.

M5-brane on M2-brane extended superspacetime - Outlook

All cocycles here are Spin-invariant. Hence we may ask for extending them from super-Minkowski to super-Poincaré $\mathfrak{iso}(\mathbb{R}^{10,1|\mathbf{32}})$.

Such extensions are given by shifting h_3 by an $\mathfrak{so}$-3-cocycle and μ_7 by an $\mathfrak{so}$-7-cocycle. The only such are $\propto \langle \omega^{\wedge 3}\rangle$, $\langle \omega^{\wedge 7}\rangle$:

$$
\begin{tikzcd}
\mathfrak{iso}(\mathbb{R}^{10,1|\mathbf{32}}) \arrow[drr, "(h_3+\langle\omega^{\wedge 3}\rangle)\wedge(g_4+\mu_4)+\frac{1}{15}\mu_7+\langle\omega^{\wedge 7}\rangle"] & & \\
\mathbb{R}^{10,1|\mathbf{32}} \arrow[u] \arrow[rr, "h_3\wedge(g_4+\mu_4)+\frac{1}{15}\mu_7"'] & & b^6\mathbb{R}/b^2\mathbb{R}
\end{tikzcd}
.
$$

This is then to be globalized not just over X, but over the frame bundle $\mathrm{Fr}(X)$. By the above obstruction theory, the parameterization of the WZW terms for $\langle\omega^{\wedge 3}\rangle$ and $\langle\omega^{\wedge 7}\rangle$ over the Frame bundle imposes String-structure and Fivebrane structure (cancelling the I_8-one loop term).

Conclusion

- There is good general abstract theory for prequantized definite globalizations of higher degree forms over higher étale stacks.
- Higher Lie theory provides prequantization of every L_∞-cocycle to a higher WZW term.
- Applying this to the bouquet of cocycles emanating from the superpoint yields super-orbifolds equipped with Lorentzian structure solving the vacuum Einstein equations of 11-dimensional supergravity and equipped with the classical anomaly cancellation that makes the M2-brane and M5-brane sigma models globally well-defined. The group stack of symmetries of these structures encodes various torsion corrections to the BPS charge extension of the superisometries.

The restriction to *vacuum* solutions (vanishing gravitino and C-field strength) could be circumvented by intervening by hand, but it is maybe noteworthy that these are the solutions relevant for realistic phenomenology (e.g. Acharya'02, Acharya'12).

Thank you!

For more details see course notes at

ncatlab.org/schreiber/show/Structure+Theory+for+Higher+WZW+Terms

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