

stochastic models for real-world processes

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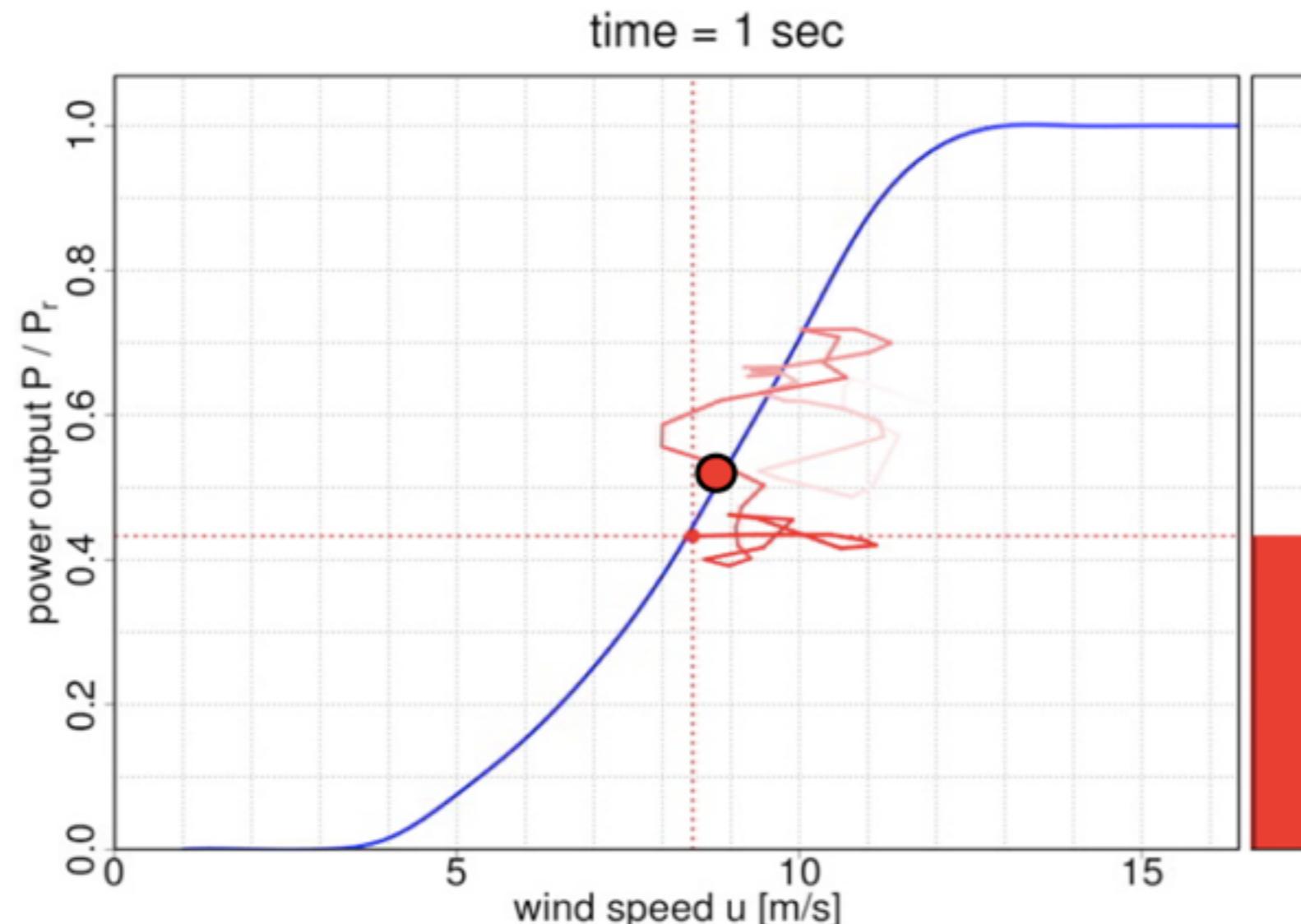
motivation

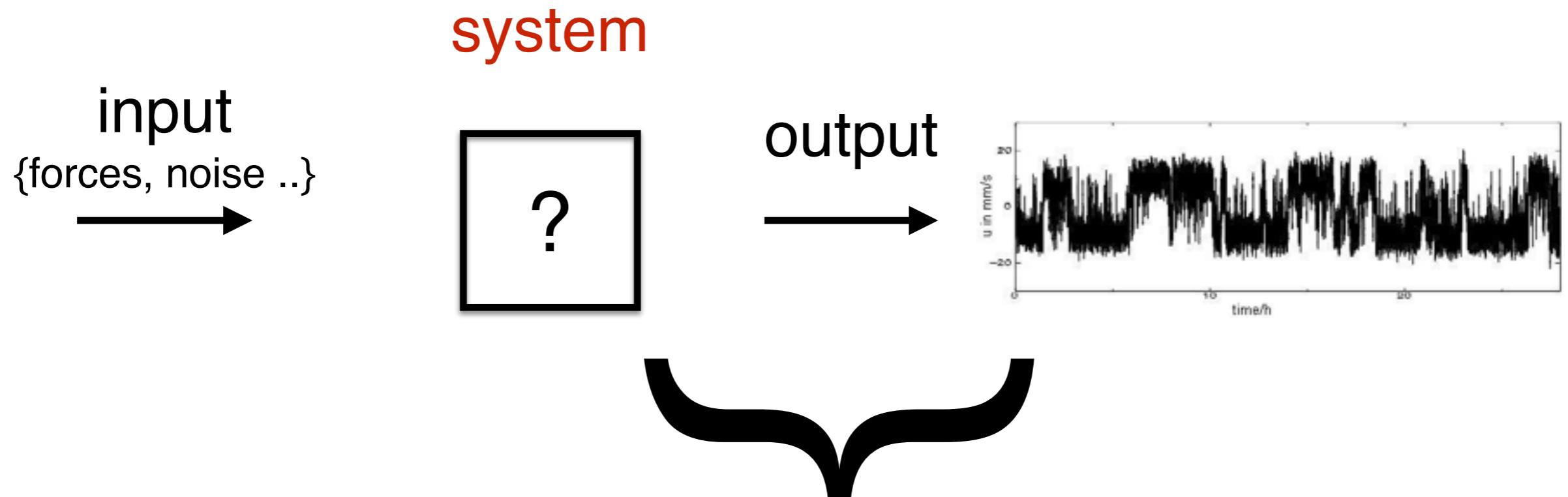
WEC >5MW



dynamics of power conversion

$$P_{WT} = \frac{1}{2} c_p(\lambda) \rho u_{wind}^3 \cdot A$$





we want to know:

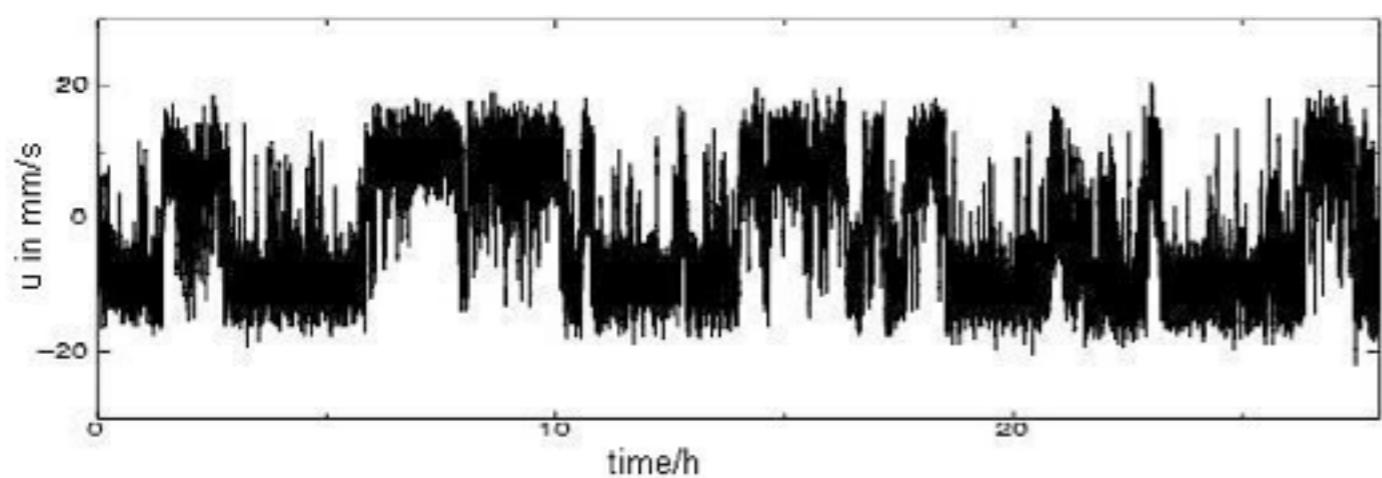
- model
- characterization
- prediction

$$d\mathbf{x} = \mathbf{D}^{(1)}(\mathbf{x}, t) dt + \sqrt{\mathbf{D}^{(2)}(\mathbf{x}, t)} dW$$

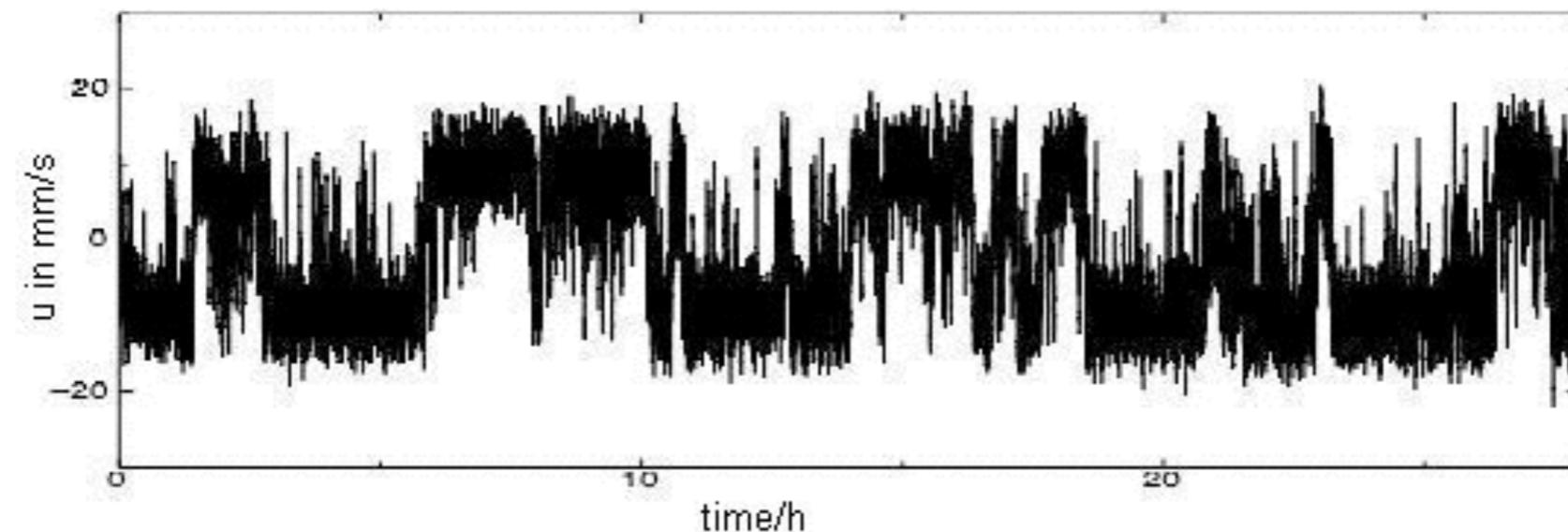
▼ stochastic processes - ideal tool for experimentalist

▼ time dependent complexity

▼ scale dependent complexity



Complex systems driven by noise and nonlinearity



open question - what is the corresponding dynamics

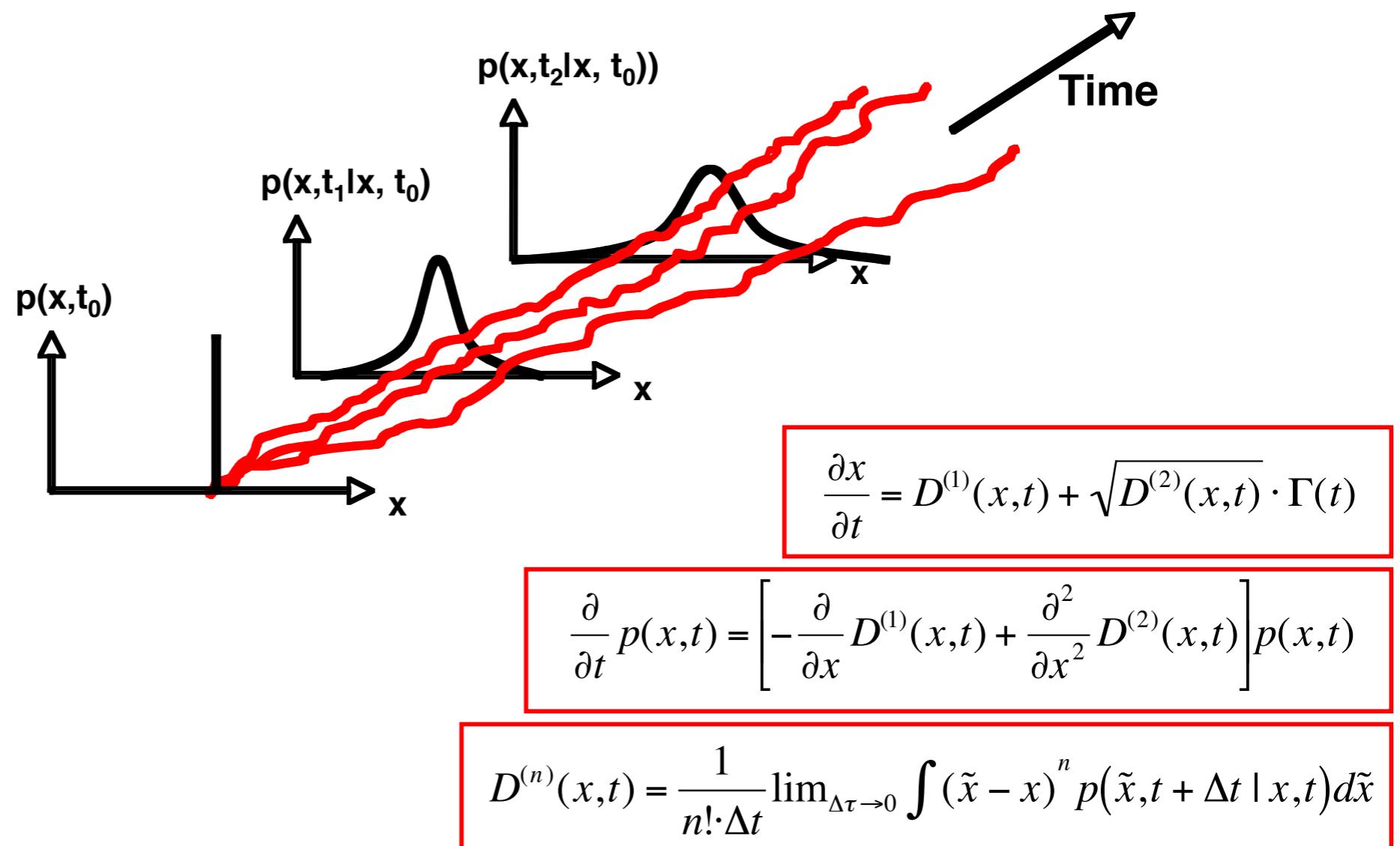
$$\dot{\mathbf{x}} = ??$$

$$\mathbf{x}(t + \tau) = ??$$

Langevin- and Kolmogorov (Fokker-Planck) Equation

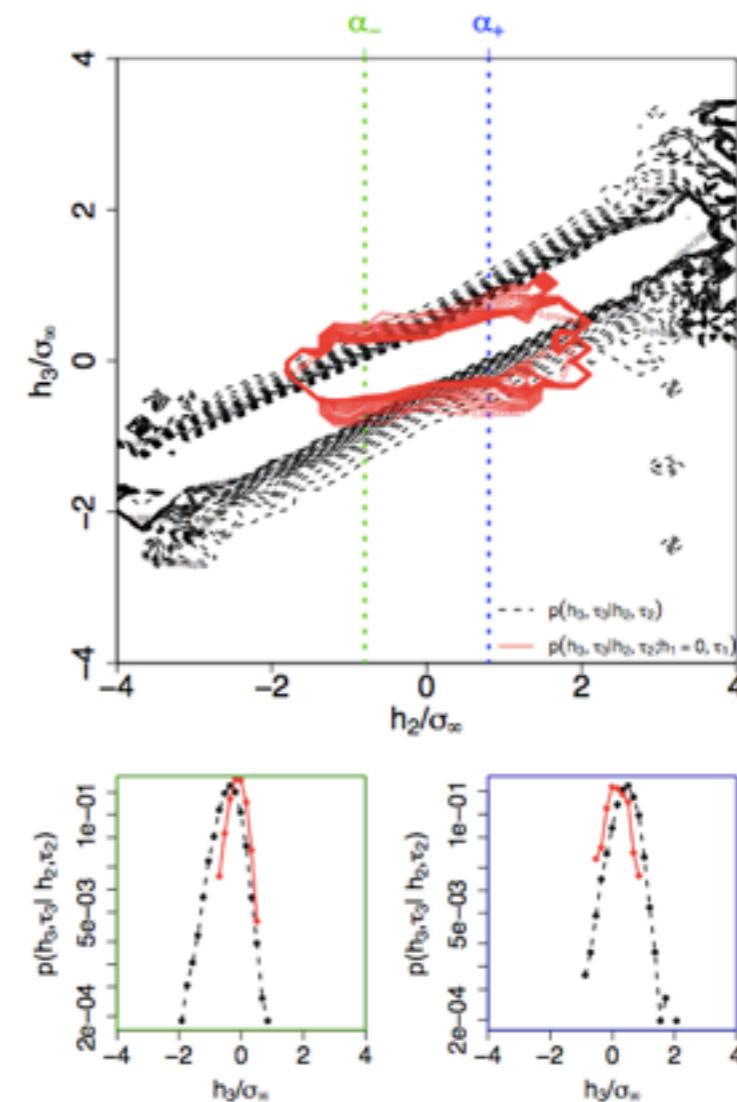
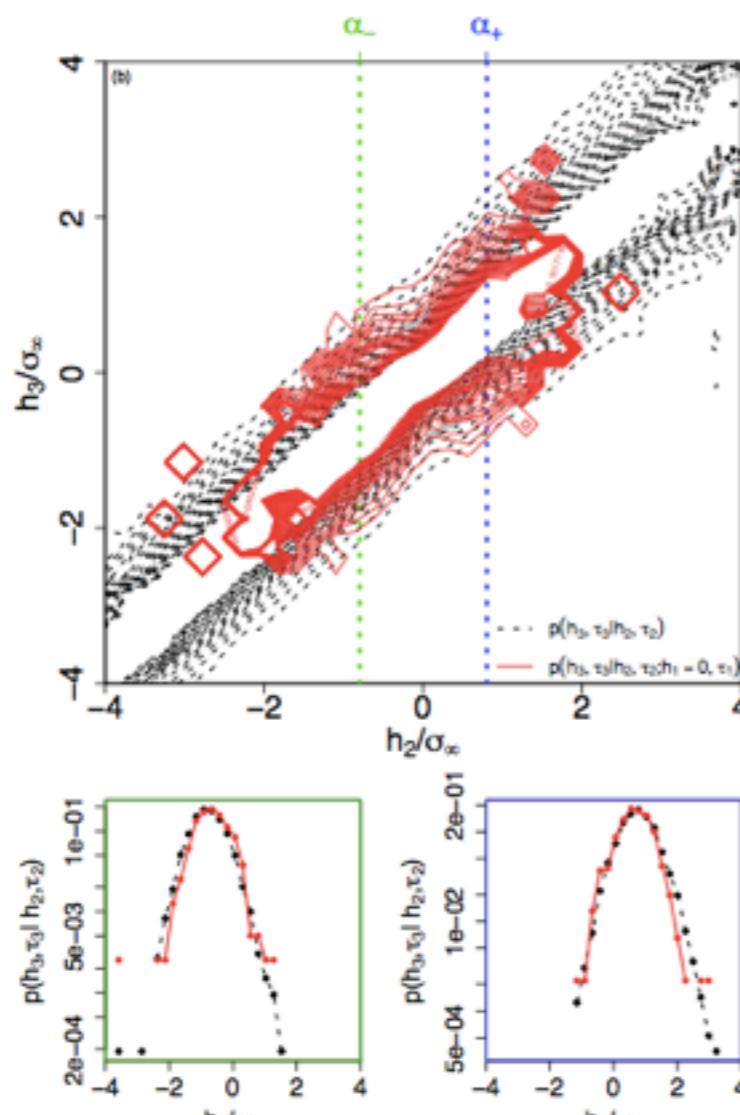
for stochastic description:

- **Markow properties** (process without memory - or delta correlated noise))



test of Markow property

$$p(\mathbf{x}_3, t_3 | \mathbf{x}_2, t_2; \mathbf{x}_1, t_1; \dots) = p(\mathbf{x}_3, t_3 | \mathbf{x}_2, t_2)$$



New Journal of Physics 16, 053037 (2014)

Einstein Markow length

A. Einstein Ann. Phys. 17, 549 (1905)

5. Über die von der molekularkinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen;
von A. Einstein.

§ 4. Über die ungeordnete Bewegung von in einer Flüssigkeit suspendierten Teilchen und deren Beziehung zur Diffusion.

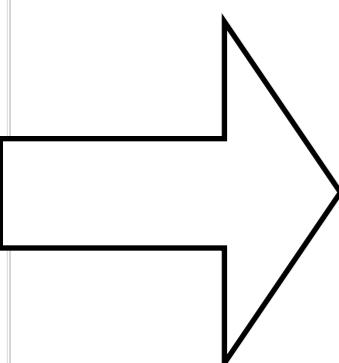
Wir gehen nun dazu über, die ungeordneten Bewegungen genauer zu untersuchen, welche, von der Molekularbewegung der Wärme hervorgerufen, Anlaß zu der im letzten Paragraphen untersuchten Diffusion geben.

Es muß offenbar angenommen werden, daß jedes einzelne Teilchen eine Bewegung ausführe, welche unabhängig ist von der Bewegung aller anderen Teilchen; es werden auch die Bewegungen eines und desselben Teilchens in verschiedenen Zeitintervallen als voneinander unabhängige Vorgänge aufzufassen sein, solange wir diese Zeitintervalle nicht zu klein gewählt denken.

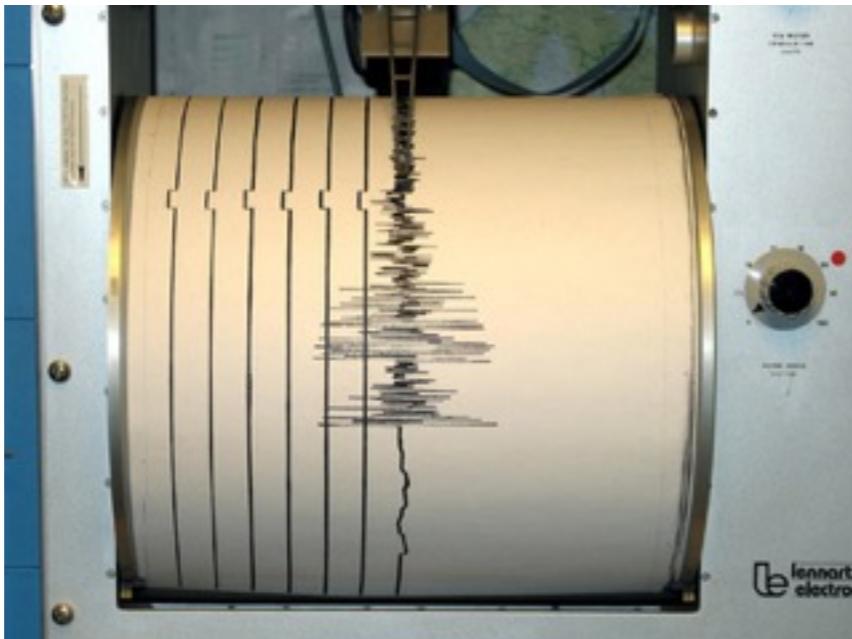
Wir führen ein Zeitintervall τ in die Betrachtung ein, welches sehr klein sei gegen die beobachtbaren Zeitintervalle, aber doch so groß, daß die in zwei aufeinanderfolgenden Zeitintervallen τ von einem Teilchen ausgeführten Bewegungen als voneinander unabhängige Ereignisse aufzufassen sind.

for sufficient large time steps the process may become Markowian

- Einstein-Markow length



Einstein Markow length - example: earthquakes



Zerstörte Infrastruktur, Erdbeben 1995, Kobe



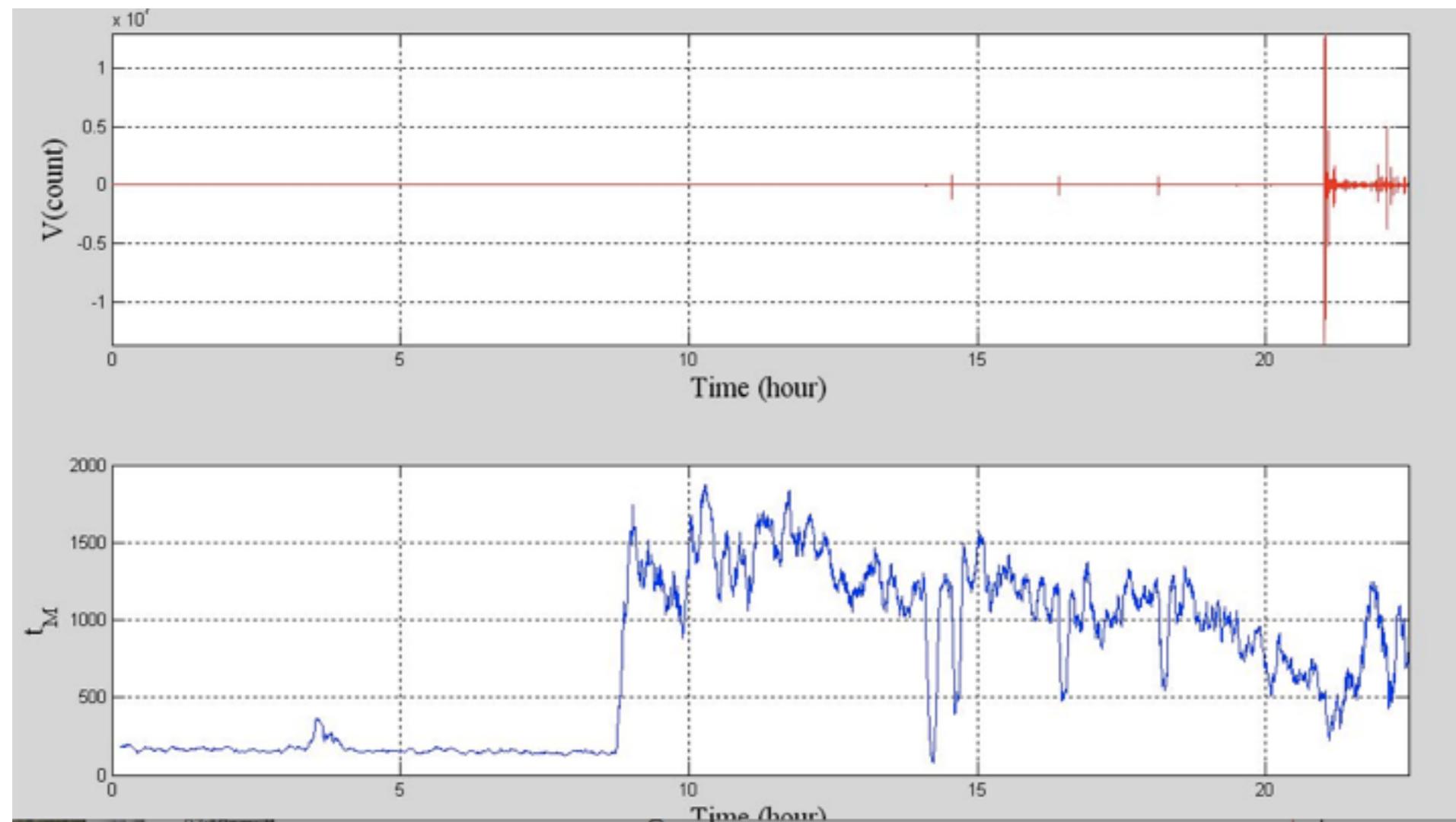
06.04.2009 - Erdbeben in Italien: Helfer transportieren einen Verletzten nach dem schweren Erdbeben in der italienischen

Einstein Markow length



06.04.2009 - Erdbeben in Italien: Helfer transportieren einen Verletzten nach dem schweren Erdbeben in der italienischen

▼ Einstein Markow length for Italy

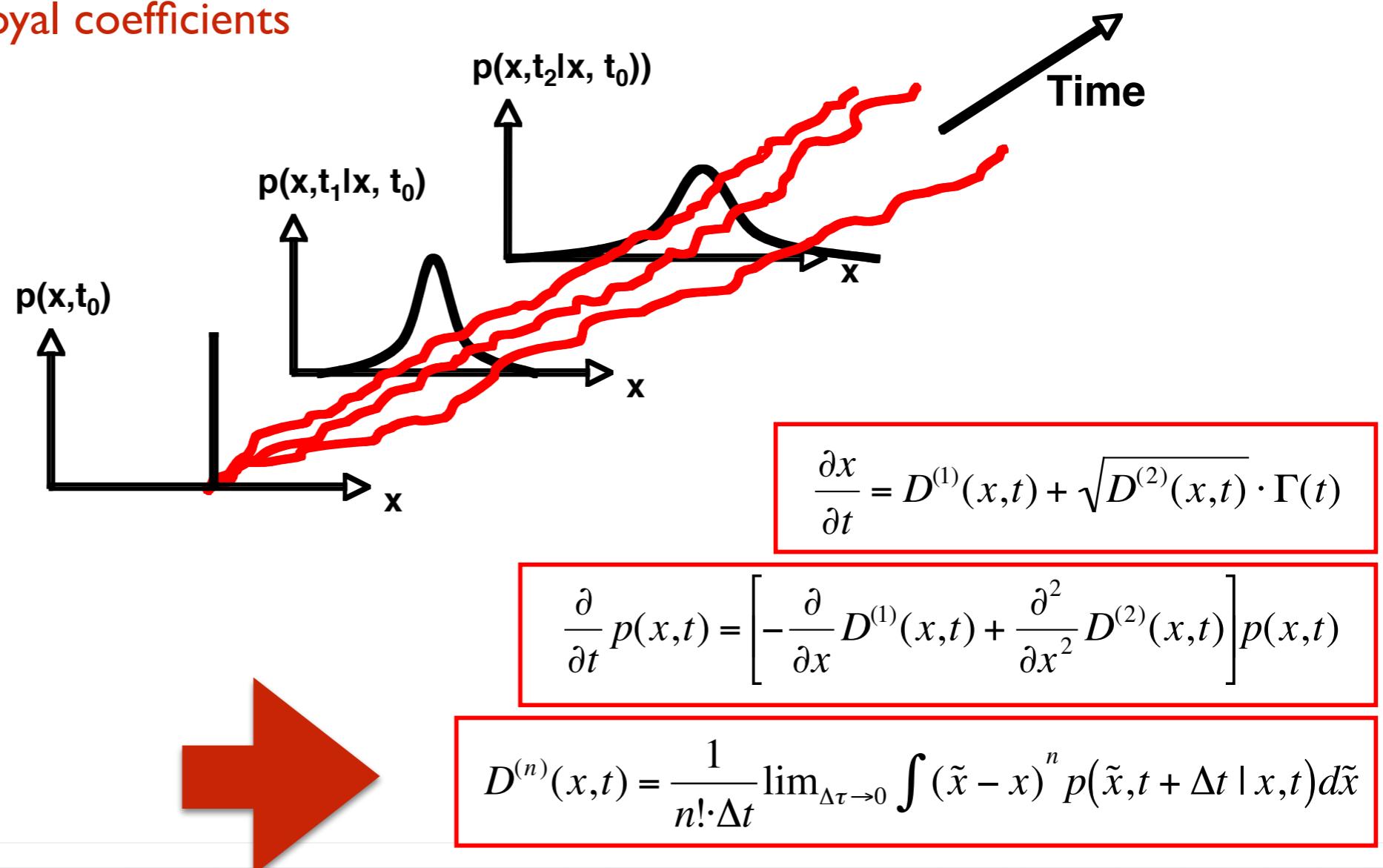


M.R.R. Tabar, et.al. Lecture Notes in Physics , Vol. 705, (Springer, 2006) 281-301.

Langevin- and Kolmogorov (Fokker-Planck) Equation

for stochastic description:

- Markow properties (process without memory - or delta correlated noise))
- Reduction to Kol. equation (Pawula Theorem - $D^{(4)} = 0$ or Gaussian noise)
- estimation of Kramers, Moyal coefficients



▼ example for D1 estimation

$$\mathbf{D}^{(1)}(\mathbf{x}) = \lim_{\tau \rightarrow 0} \frac{1}{\tau} \left\langle \mathbf{X}(t + \tau) - \mathbf{x} \right\rangle \Big|_{\mathbf{X}(t) = \mathbf{x}}$$

$$\mathbf{D}^{(n)}(\mathbf{x}) = \lim_{\tau \rightarrow 0} \frac{1}{n! \cdot \tau} \int (\tilde{x} - x)^n p(\tilde{x}, t + \tau | x, t) d\tilde{x}$$

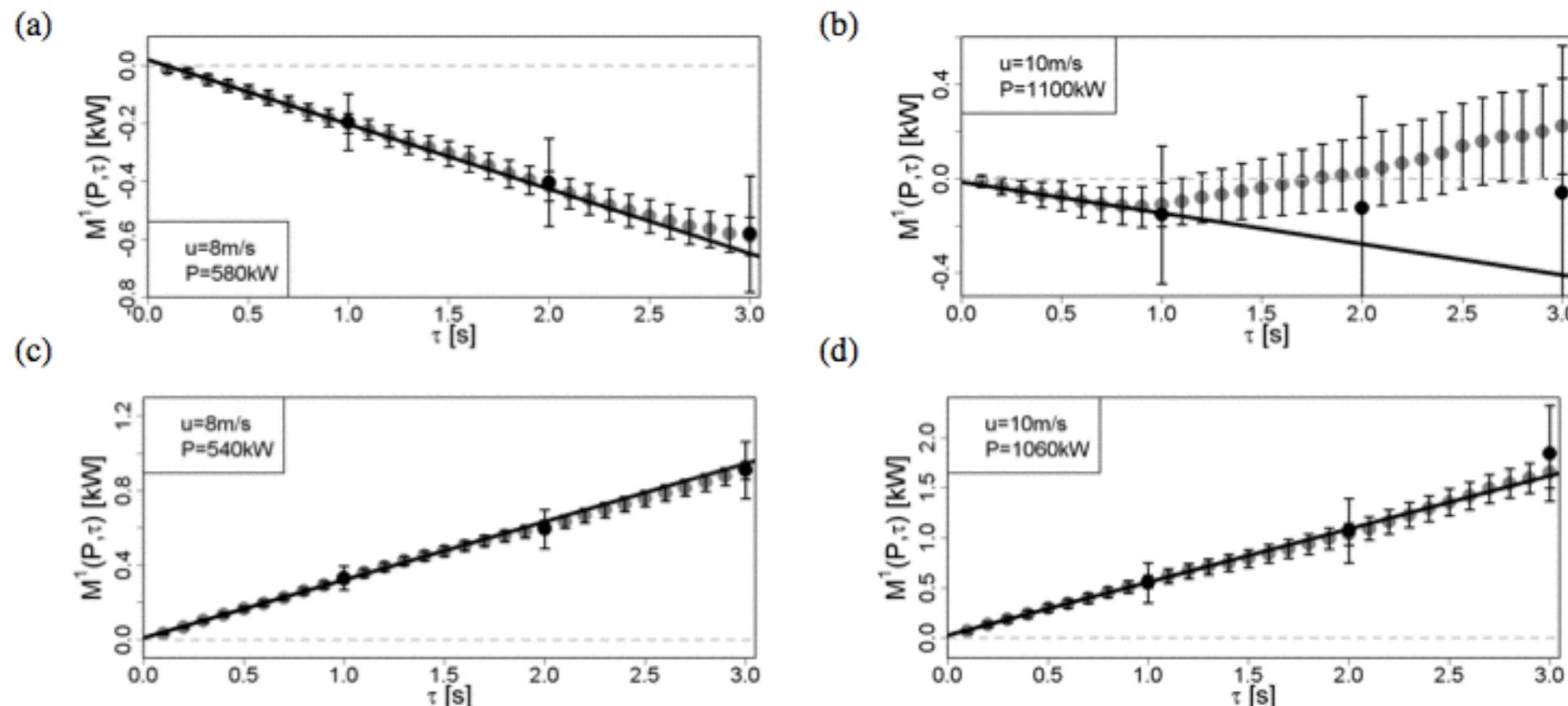


Figure 2. Evolution of the conditional moments of *data5* within small τ values for (a), (c) $u=8\pm0.25\text{ m/s}$ (superposed zoom of figure 1) and (b), (d) $10\pm0.25\text{ m/s}$. The black line depicts the linear fit of the 10 Hz data in the range of $\tau_{10} \in [0.3\text{s}, 0.8\text{s}]$, respectively.

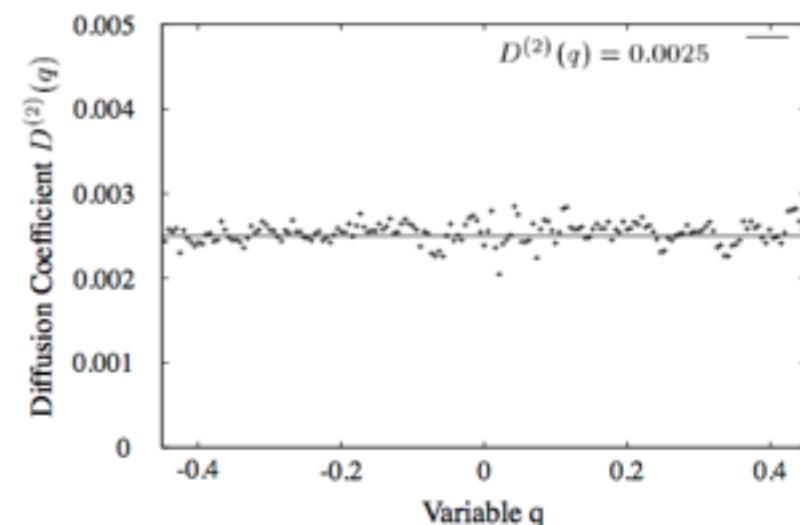
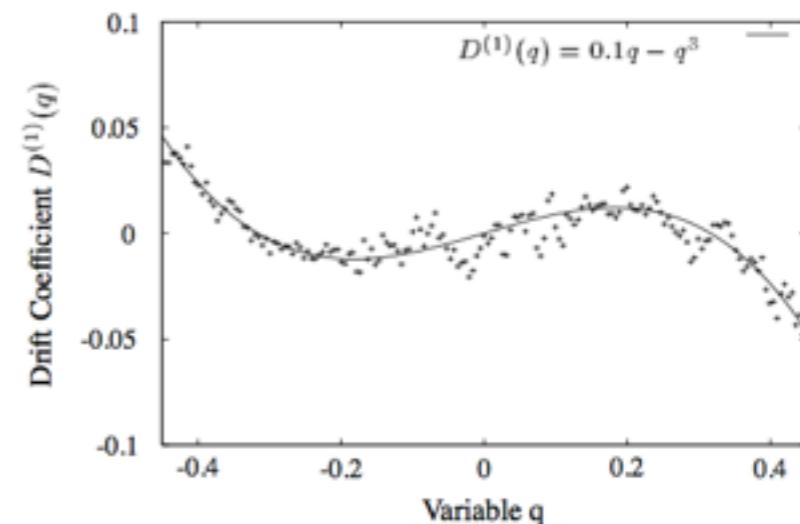
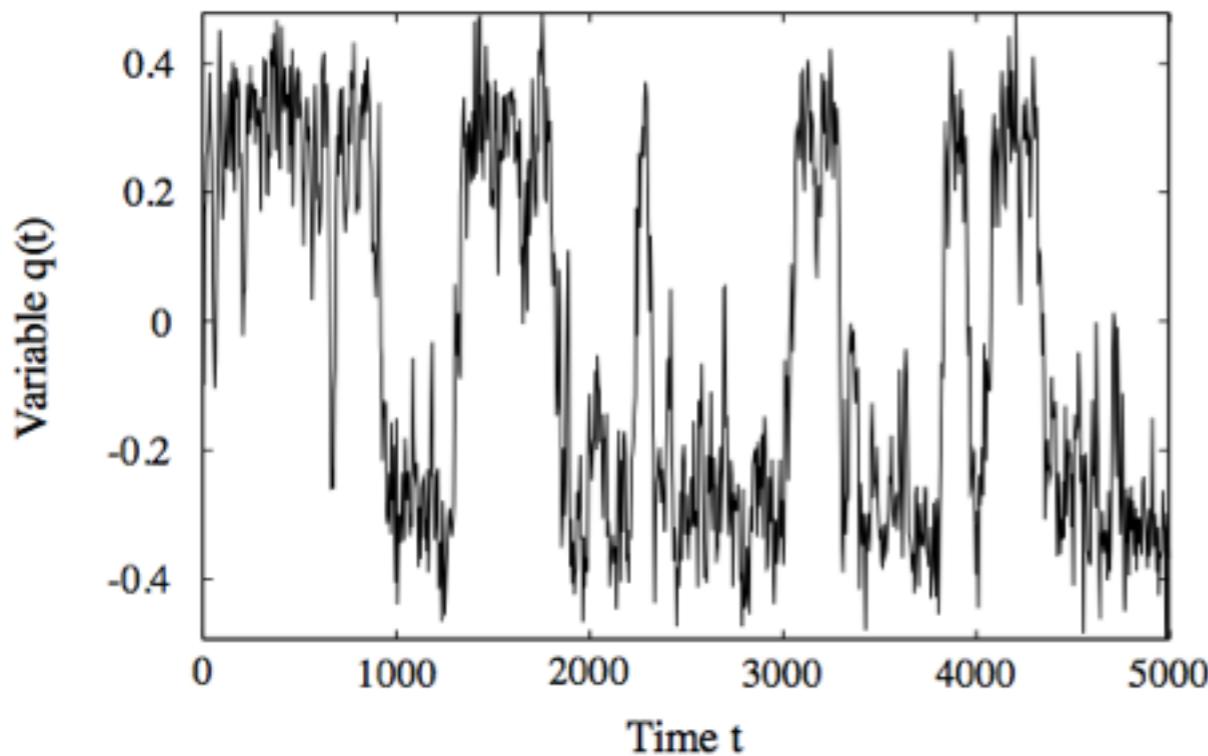
first paper

S. Siegert, R. Friedrich, and J. Peinke :
Analysis of Data of Stochastic Systems
Phys. Lett. A **243**, 275 (1998)

reconstructed drift and diffusion

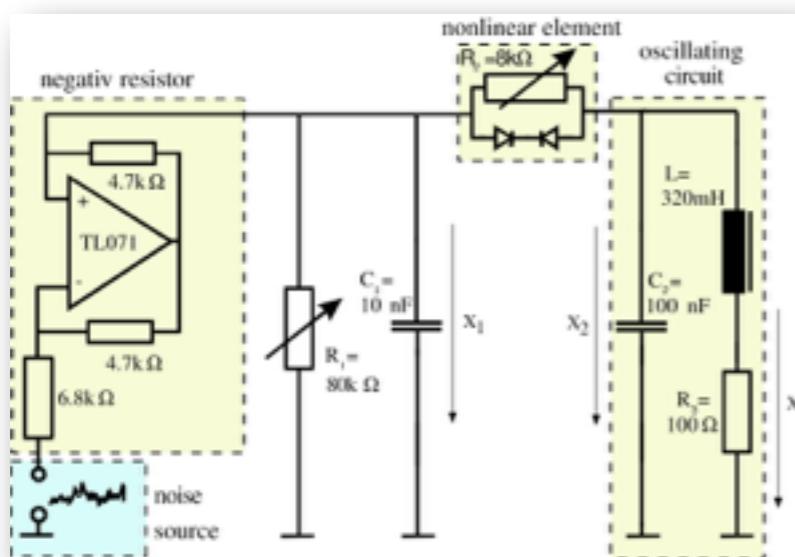
numeric model

$$\dot{q}(t) = 0.1q(t) - q^3(t) + 0.05 \cdot F(t),$$

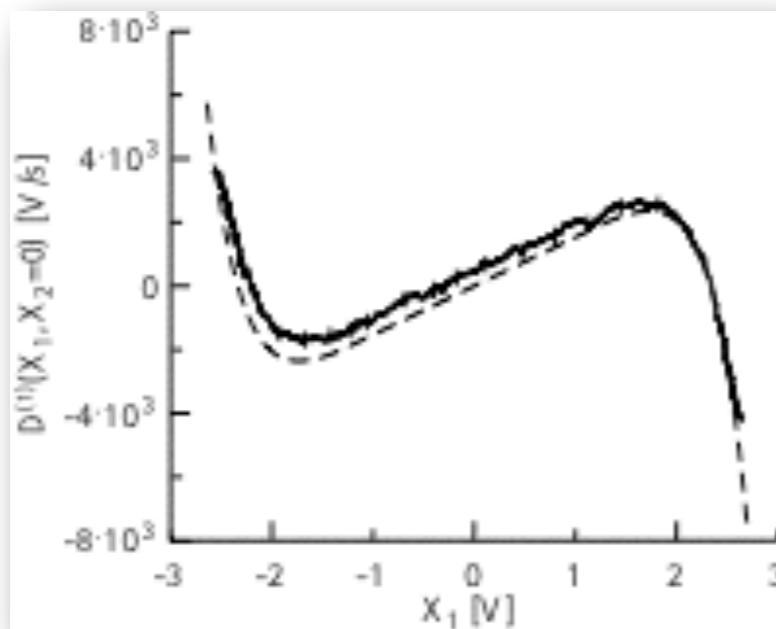


further applications - chaotic dynamics

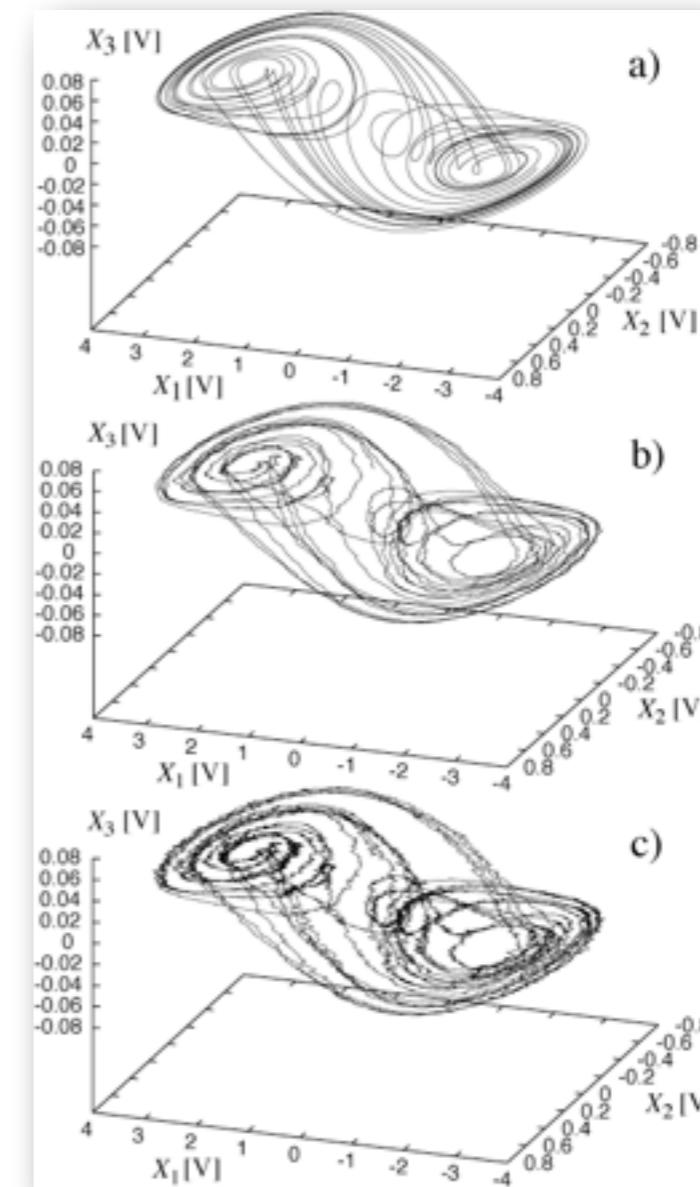
Shinriki Oszillator



reconstructed dynamics



chaotic attractors with
and without noise



Europhys. Lett. 61, 466 (2003)

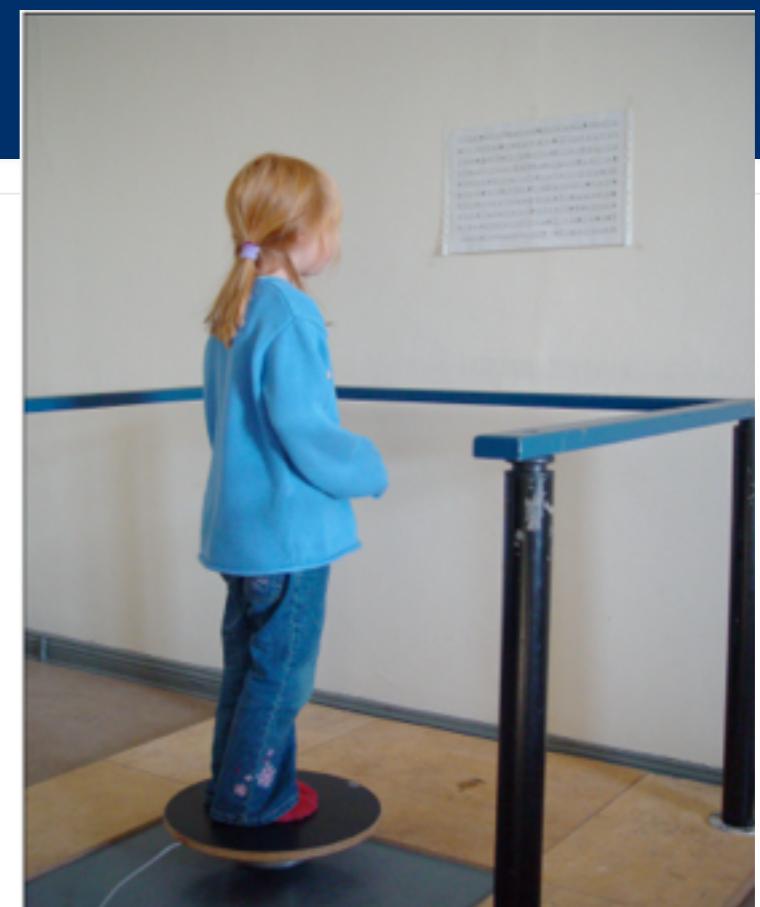
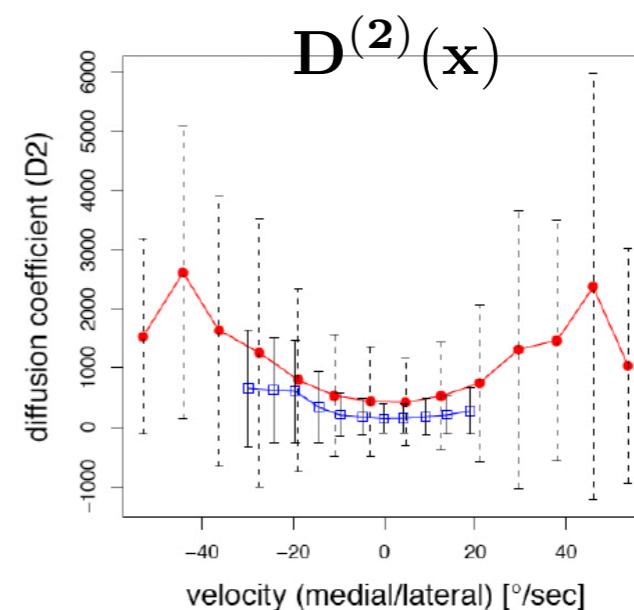
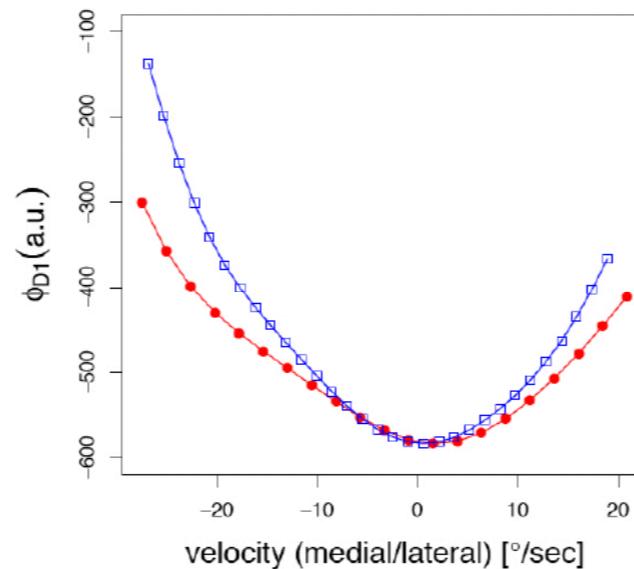
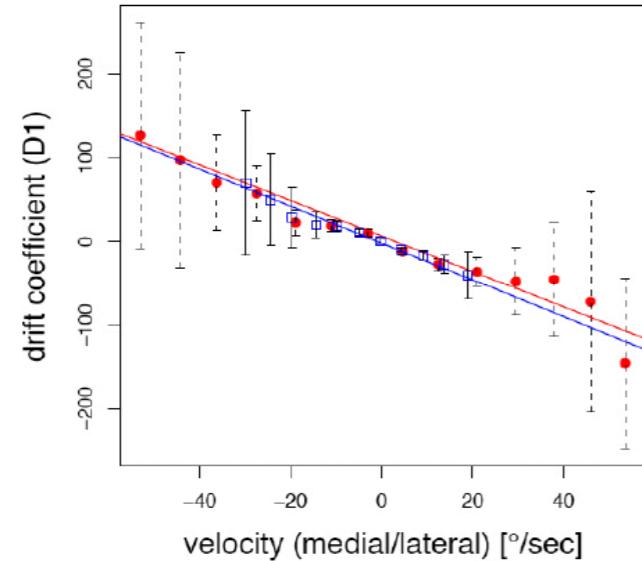
Balance

analysis of the dynamics of balance
with and without task

(cooperation with V. Lippens

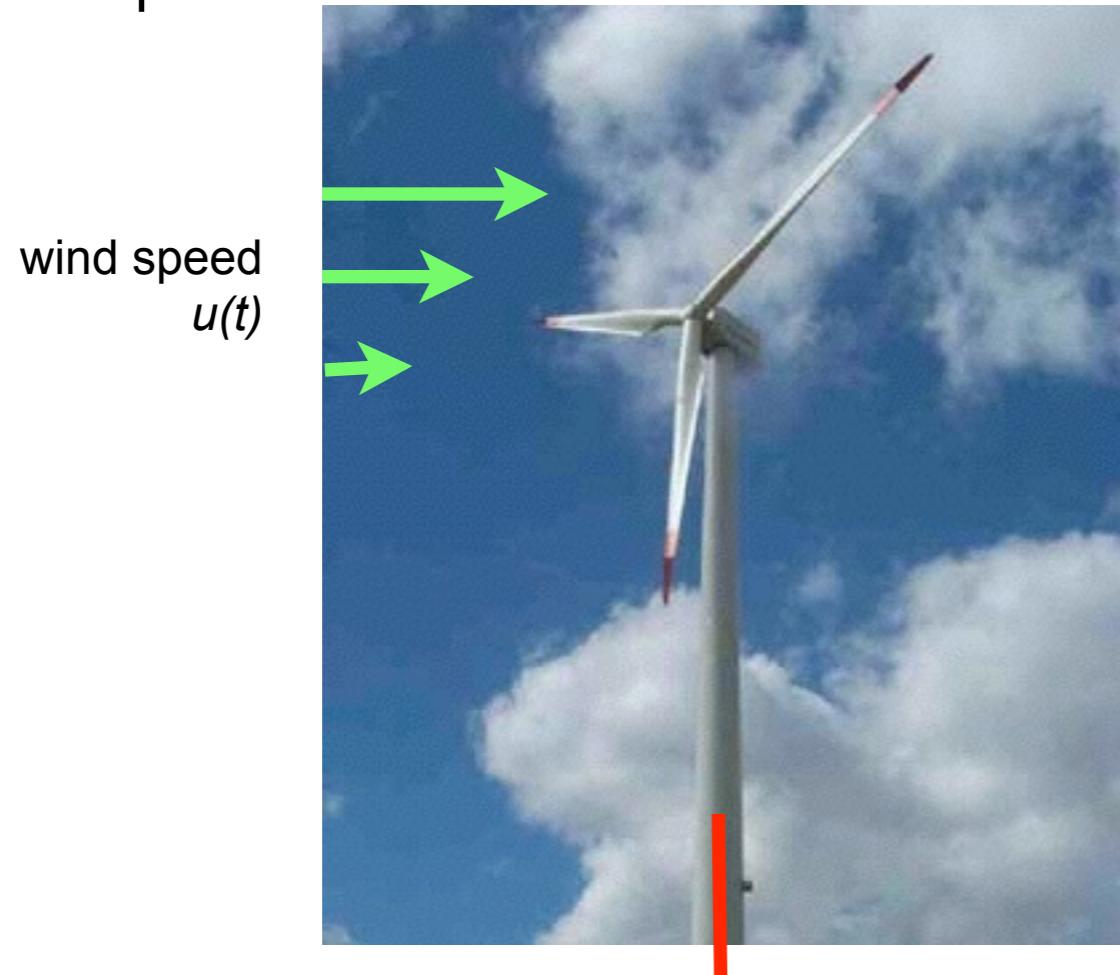
Physics Letters A 373, 811 (2009))

$$\Phi(X) = - \int_{-\infty}^X dx' \mathbf{D}^{(1)}(\mathbf{x}')$$



wind turbine

input: turbulent Wind
dynamic power curve

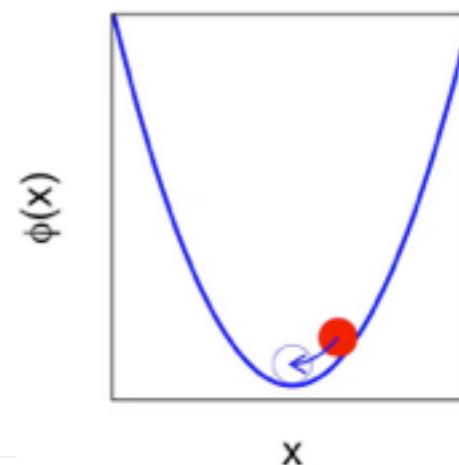
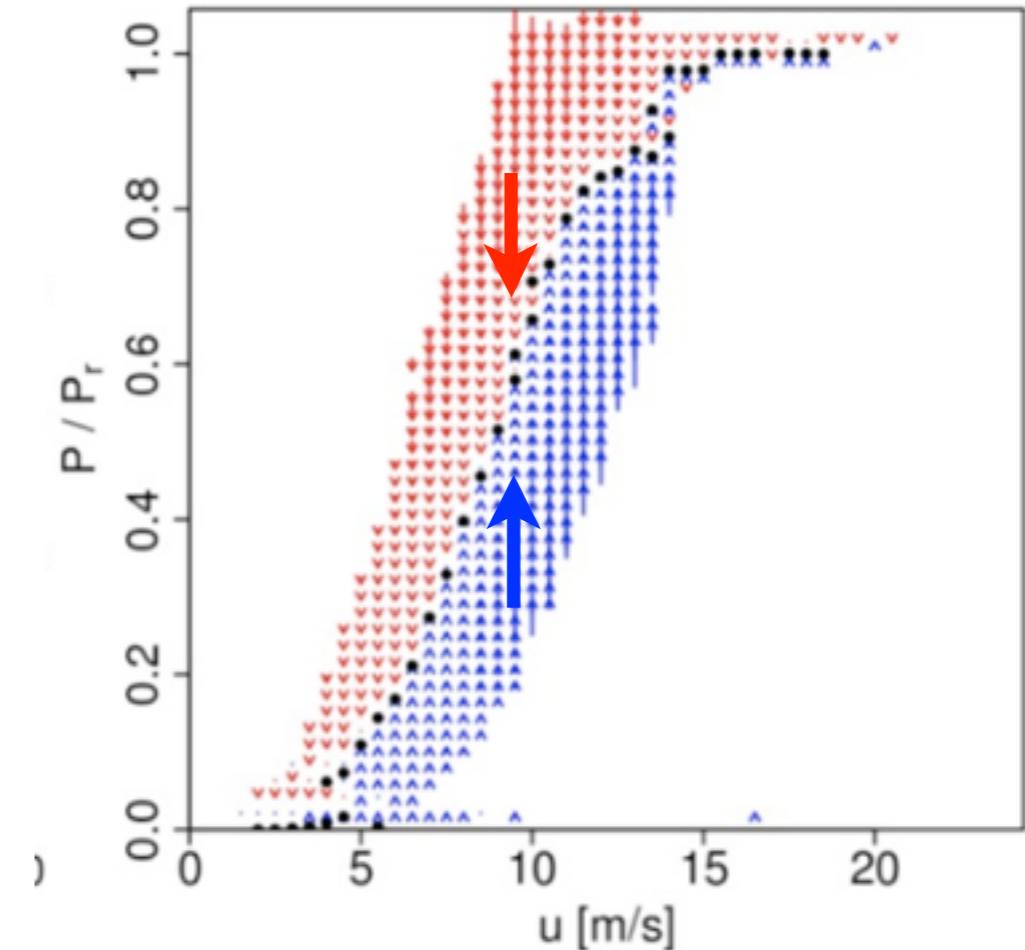
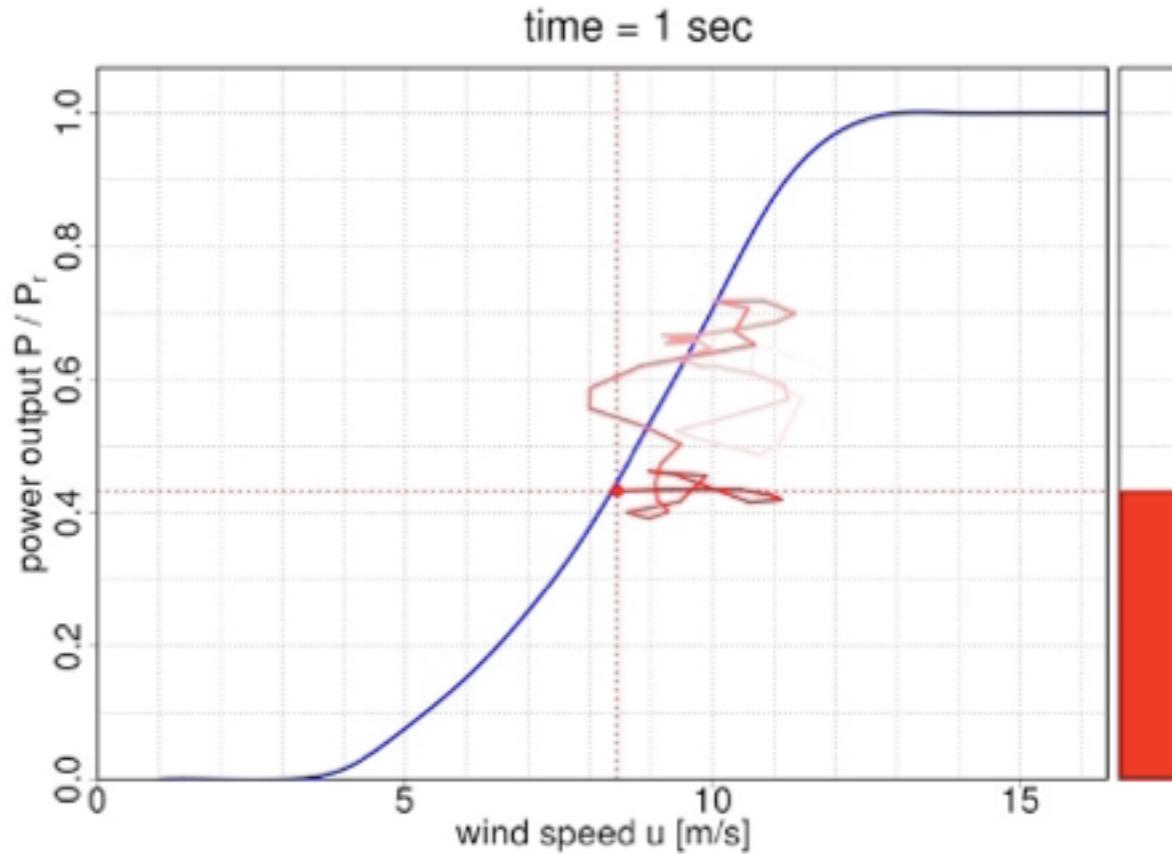


power - dynamic response

conversion dynamics given by
aerodynamics and control
system



stochastic motion in a potential



summary - time dependent systems

typical problem arising for experimental data:

$$d\mathbf{x} = \mathbf{D}^{(1)}(\mathbf{x}, t) dt + \sqrt{\mathbf{D}^{(2)}(\mathbf{x}, t)} dW$$

- noise - delta correlated?

- Markow properties $p(x_1, t_1 | x_2, t_2; \dots; x_n, t_n) = p(x_1, t_1 | x_2, t_2)$

- what are the function of $\mathbf{D}^{(i)}$?
 - Kramers-Moyal coefficient

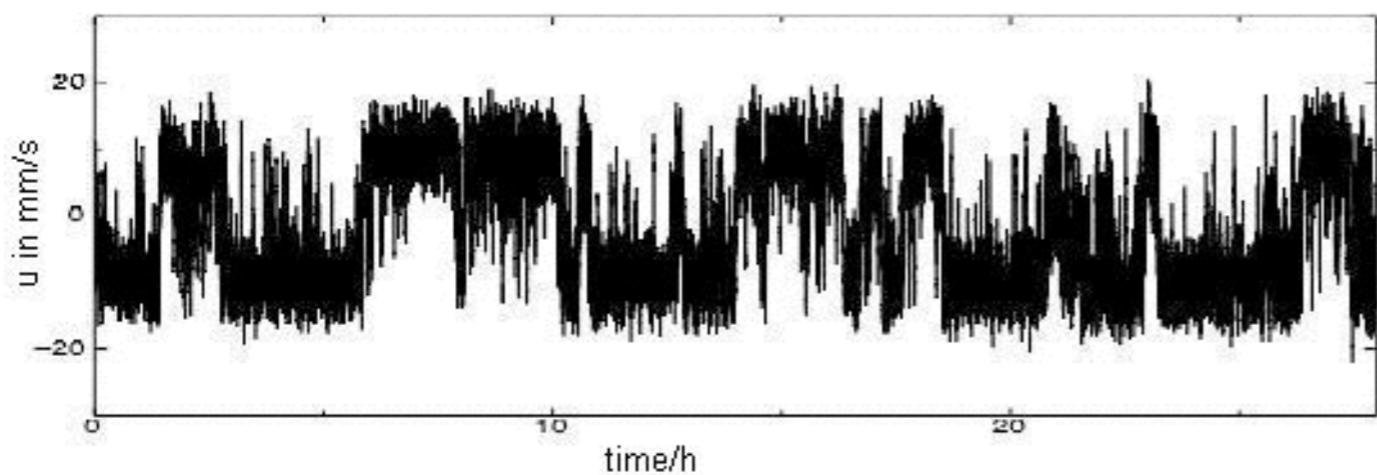
$$\mathbf{D}^{(n)}(\mathbf{x}) = \lim_{\tau \rightarrow 0} \frac{1}{n! \cdot \tau} \int (\tilde{x} - x)^n p(\tilde{x}, t + \tau | x, t) d\tilde{x}$$

- limit small steps

$$\lim_{\tau \rightarrow 0}$$

2nd part

- stochastic processes - ideal tool for experimentalist
- time dependent complexity
- scale dependent complexity



turbulence

- ▼ comprehensive description by n-point statistics



$$\partial_t \vec{u} + (\vec{u} \cdot \nabla) \vec{u} = -\nabla p + \frac{1}{Re} \Delta \vec{u} + \vec{f}$$

$$\nabla \cdot \vec{u} = 0$$

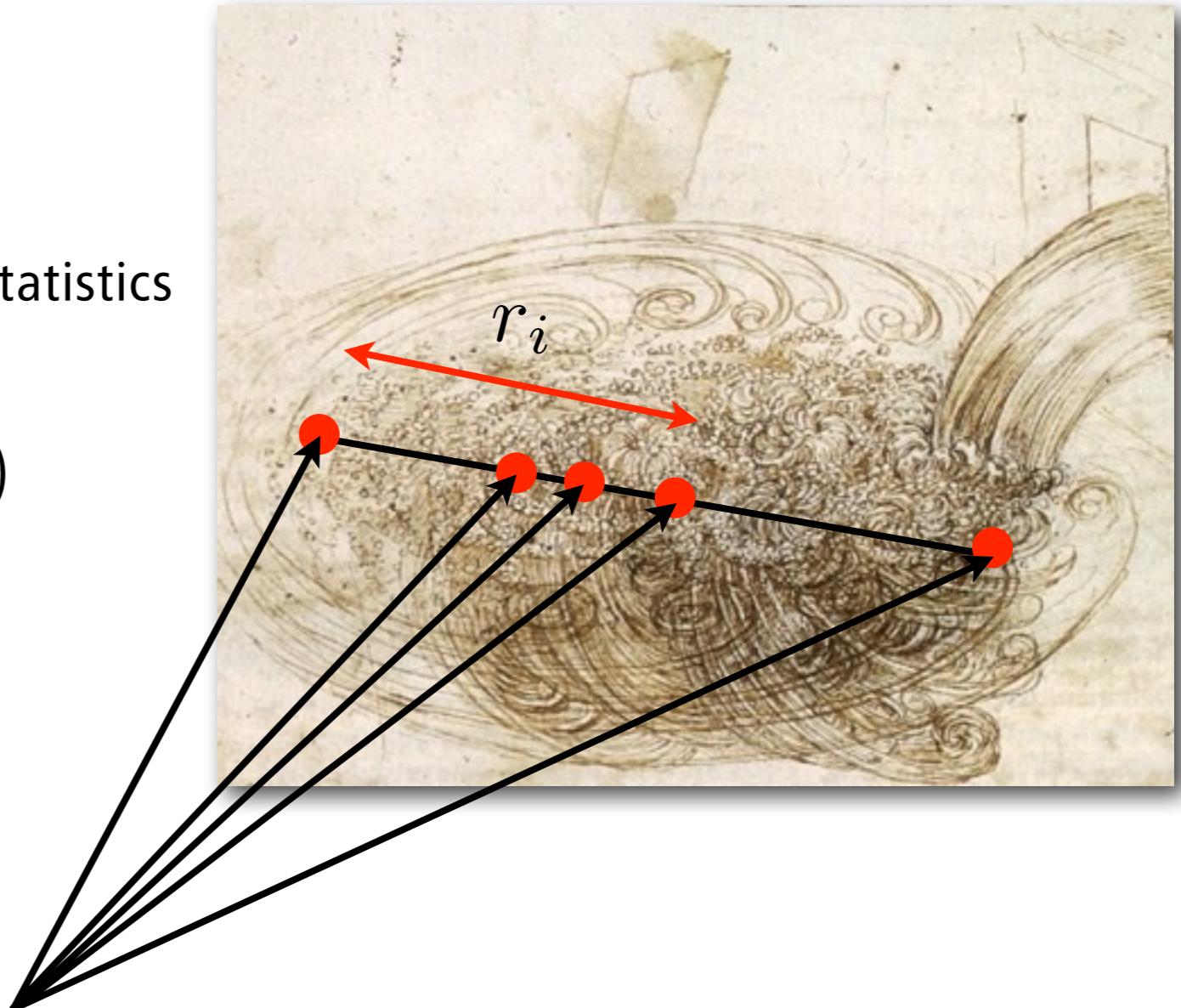
turbulence

▼ comprehensive description by n-point statistics

$$p(u(x_1), \dots, u(x_{n+1}))$$

using velocity increments:

$$u_{r_i} = u(x + r_i) - u(x_i)$$



$$p(u(x_1), \dots, u(x_{n+1})) = p(u_{r_1}, \dots, u_{r_n}, u(x_1))$$

n-point statistics

$$\begin{aligned} p(u(x_1), \dots, u(x_{n+1})) &= p(u_{r_1}, \dots, u_{r_n}, u(x_1)) \\ &= p(u_{r_1}, \dots, u_{r_n} | u(x_1)) \cdot p(u(x_1)) \end{aligned}$$



Bayes theorem - joint pdf by cond. pdf

$$= p(u_{r_1} | \dots, u_{r_n} u(x_1)) p(u_{r_2} | \dots, u_{r_n} u(x_1)) \dots \cdot p(u(x_1))$$

can be simplified - exp evidence
is Markow in scale r



$$= p(u_{r_1} | u_{r_2}, u(x_1)) \dots p(u_{r_{n-1}} | u_{r_n}, u(x_1)) \cdot p(u(x_1))$$



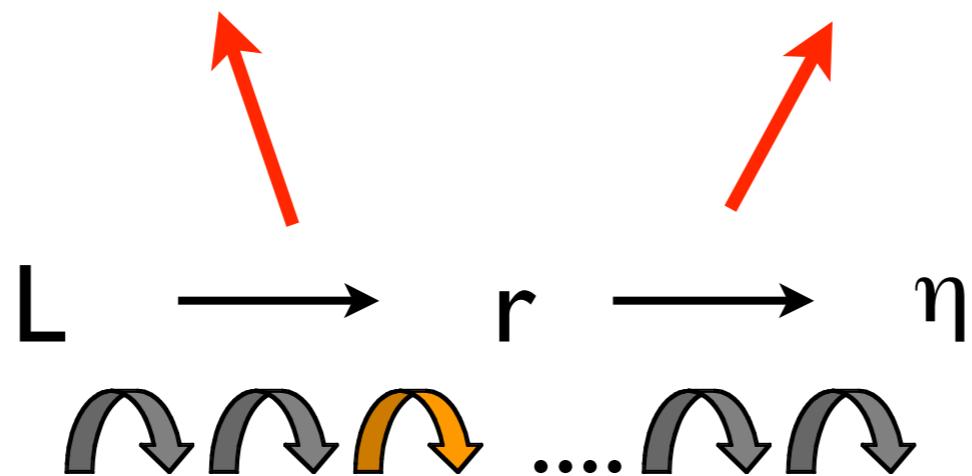
common analysis of turbulence

$$p(u_{r_1}) \dots p(u_{r_n})$$

n-point statistics

$$p(u(x_1), \dots, u(x_{n+1}))$$

$$= p(u_{r_1} | u_{r_2}, u(x_1)) \dots p(u_{r_{n-1}} | u_{r_n}, u(x_1)) \cdot p(u(x_1))$$

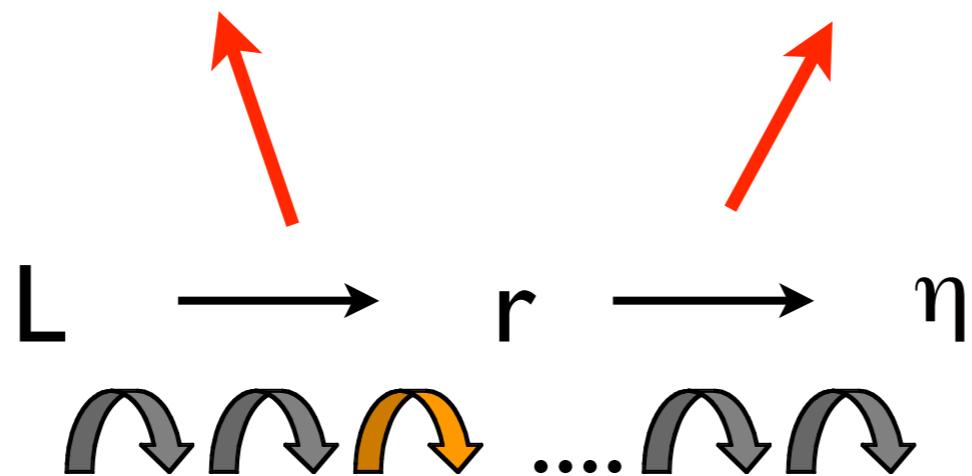


new view of cascade process:
three point closure
**local in the cascade means no memory
or Markow process in r**

n-point statistics

$$p(u(x_1), \dots, u(x_{n+1}))$$

$$= p(u_{r_1} | u_{r_2}, u(x_1)) \dots p(u_{r_{n-1}} | u_{r_n}, u(x_1)) \cdot p(u(x_1))$$



Markow prop & cascade with Fokker-Planck Equ.

$$-r_j \frac{\partial}{\partial r_j} p(u_{r_j} | u_{r_k}, u(x_1)) = \left\{ -\frac{\partial}{\partial u_{r_j}} D^{(1)}(u_{r_j}, r_j, u(x_1)) + \frac{\partial^2}{\partial u_{r_j}^2} D^{(2)}(u_{r_j}, r_j, u(x_1)) \right\} p(u_{r_j} | u_{r_k}, u(x_1))$$

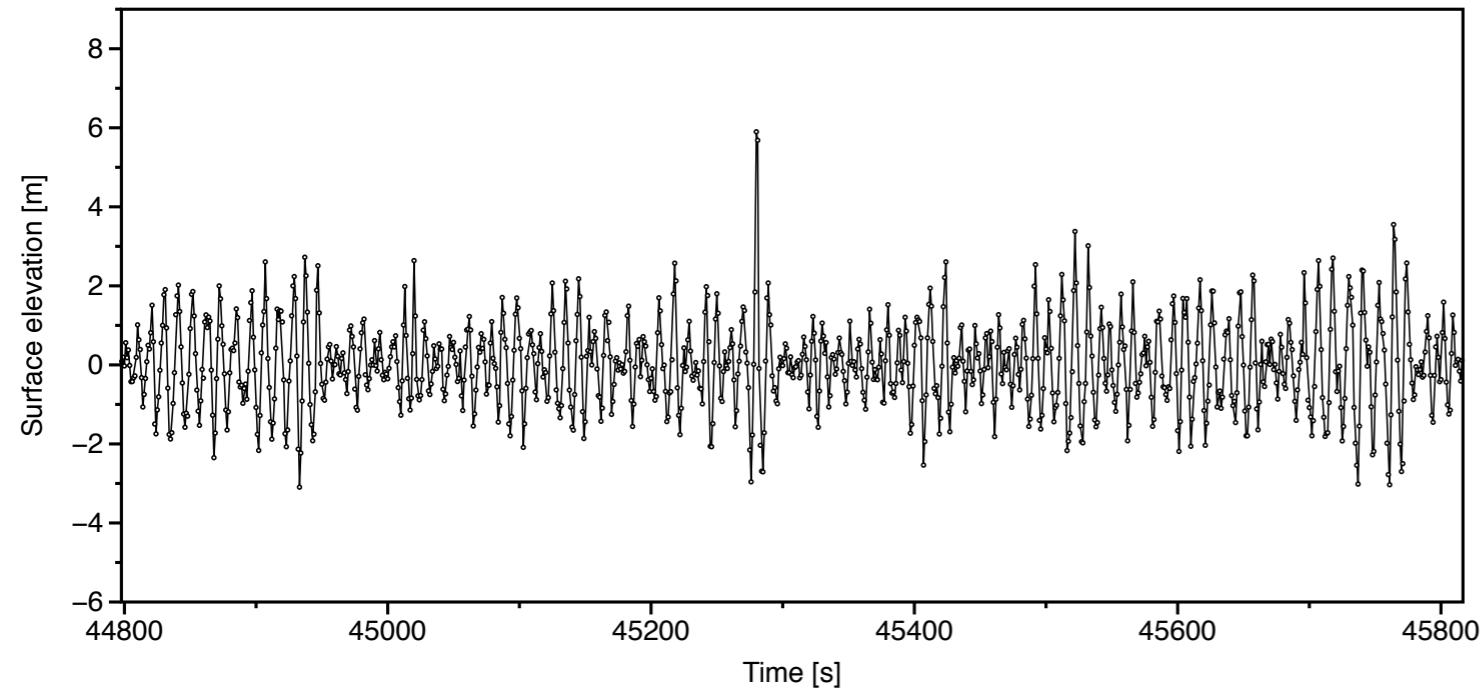
application rogue wave



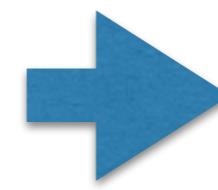
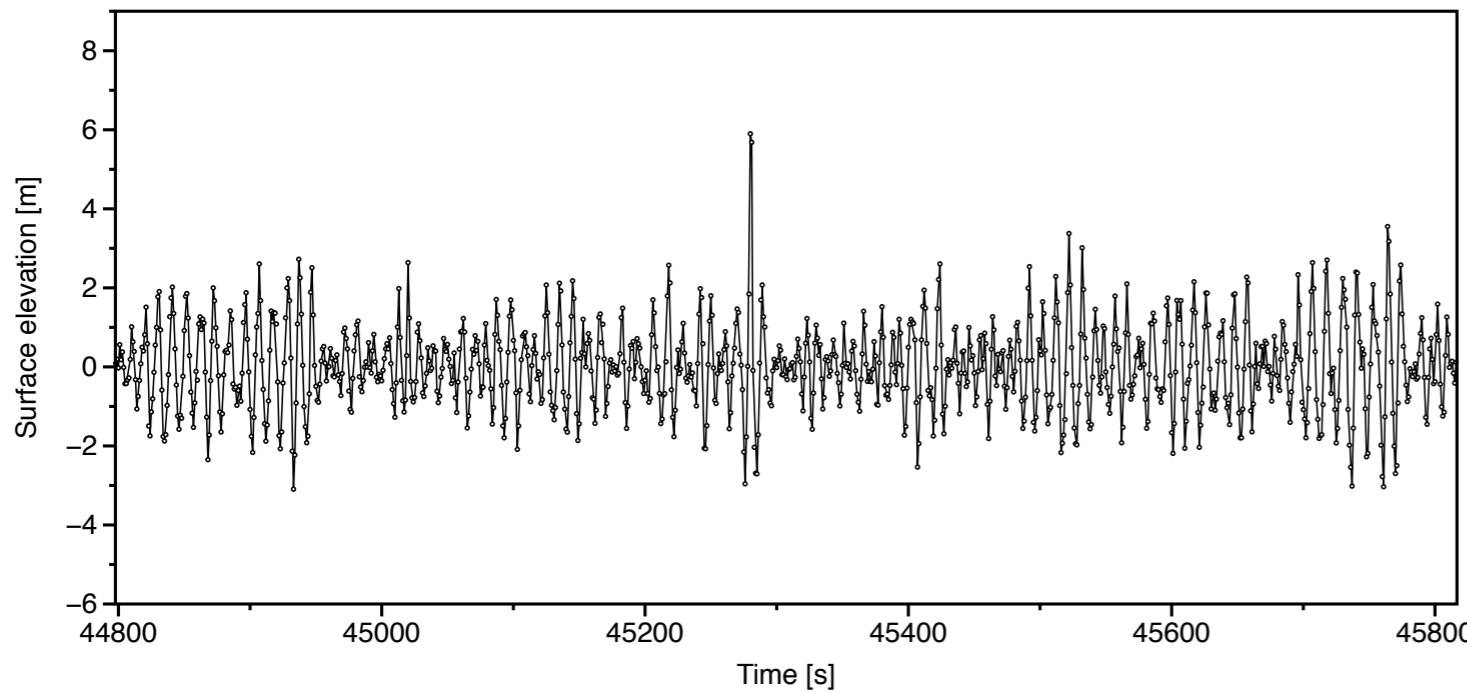
NewScientist

Environment

▼ measured data



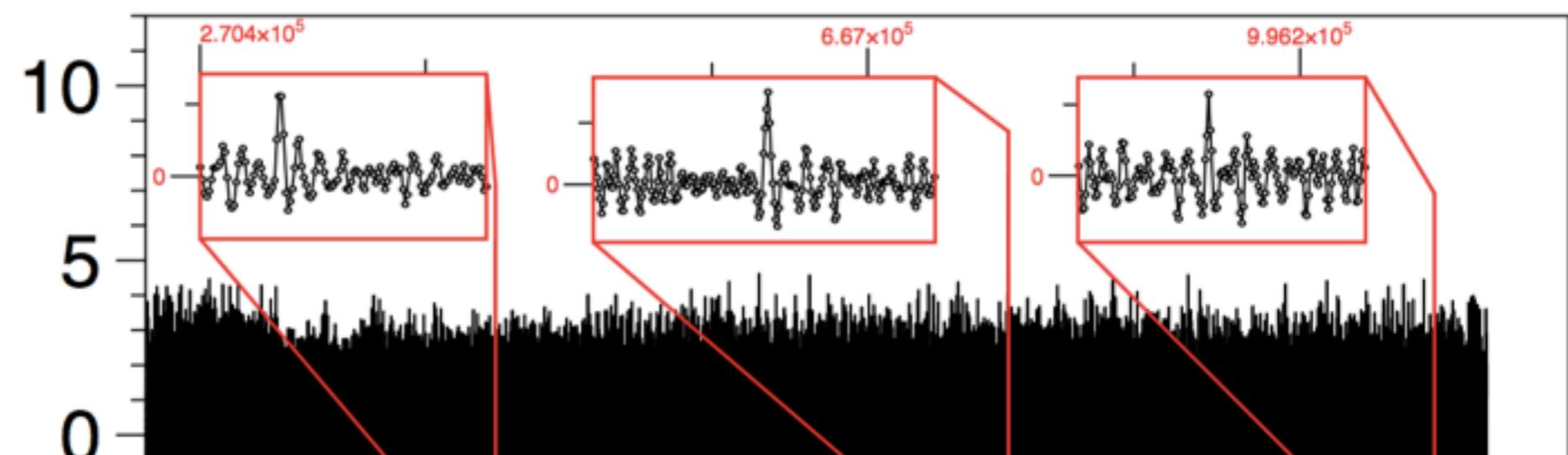
▼ measured data



stoch process



▼ from reconstructed stochastic cascade process



ams/ems/spm Porto
2015

▼ some literature

- R. Friedrich, J. Peinke, M. Sahimi and M. Reza Rahimi Tabar : Approaching Complexity by Stochastic Processes: From Biological Systems to Turbulence, Phys. Report, 506, 87-162 (2011)

- researchgate Peinke

ResearchGate

- homepage - Twist <http://www.uni-oldenburg.de/twist/>