

Stage-Structured Model for *Aedes Aegypti* and *Wolbachia* Interaction

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June 2015

THE MOSQUITO

Aedes (Stegomyia) aegypti (Diptera: Culicidae) (Linnaeus)

Tropical Areas Sub-Tropical Areas Worldwide



GEOGRAPHICAL DISTRIBUTION

Mosquito



MOSQUITO BIOLOGY



PUBLIC HEALTH ISSUE

Aedes aegypti is a vector for:

- Dengue
- Yellow Fever
- Chikungunya
- Etc.

DENGUE ELIMINATION

Prophylaxis

- No vaccine
- Mosquito population control

• Wolbachia-infected mosquitoes

POPULATION DYNAMICS MODEL



SYSTEM ODE

$$\begin{cases} \dot{Y}_{1,j} &= B(j, Y_{5,\cdot}) - \left(\tau_1^j(t) + d_1^j\right) Y_{1,j} \\ \dot{Y}_{2,j} &= \tau_1^j(t) Y_{1,j} - \left(\tau_2^j(t) + d_2^j + s^j \sum_{l=1}^4 Y_{2,l}\right) Y_{2,j} \\ \dot{Y}_{3,j} &= \tau_2^j(t) Y_{2,j} - \left(\tau_3^j(t) + d_3^j\right) Y_{3,j} \\ \dot{Y}_{4,j} &= \tau_3^j(t) Y_{3,j} - \left(\tau_4^j + d_4^j\right) Y_{4,j} \\ \dot{Y}_{5,j} &= \tau_4^j Y_{4,j} - d_5^j Y_{5,j} \end{cases}$$

where *t* is measured in days, and $Y_{i,j}$ denotes stage *i* within infected/uninfected female/male j = 1, 2, 3, 4. For instances, parous infected female $Y_{5,1}$ and parous uninfected male $Y_{5,4}$.

BIRTH FUNCTION

$$\begin{split} \mathcal{B}(j,Y_{5,\cdot}) &= Y_{5,1}\left(\frac{Y_{5,2}}{\sum_{k=1}^{2}Y_{5,2k}}\right)(SR_{1,j})(br_1)(1-cbr_1)(VT_{1,j})(1-ci_1)\\ &+ Y_{5,1}\left(\frac{Y_{5,4}}{\sum_{k=1}^{2}Y_{5,2k}}\right)(SR_{2,j})(br_2)(1-cbr_2)(VT_{2,j})(1-ci_2)\\ &+ Y_{5,3}\left(\frac{Y_{5,2}}{\sum_{k=1}^{2}Y_{5,2k}}\right)(SR_{3,j})(br_3)(1-cbr_3)(VT_{3,j})(1-ci_3)\\ &+ Y_{5,3}\left(\frac{Y_{5,4}}{\sum_{k=1}^{2}Y_{5,2k}}\right)(SR_{4,j})(br_4)(1-cbr_4)(VT_{4,j})(1-ci_4) \end{split}$$

where

$$(SR_n, j) = (sr_n, j)$$
 for $j = 1, 3$;
 $(SR_n, j) = (1 - sr_n, j)$ for $j = 2, 4$.
 $(VT_n, j) = (vt_n, j)$ for $j = 1, 2$;
 $(VT_n, j) = (1 - vt_n, j)$ for $j = 3, 4$.
 $br_n = 4$

ESTABLISHMENT OF BASIC INITIAL CONDITIONS

Assume the following non-infected population distribution:

Quantity	Value	Fraction of Total	Description
Y ₁	39,160,000	0.979	Total number of eggs
Y ₂	240,000	0.006	Total number of larvae
Y ₃	40,000	0.001	Total number of pupae
Y_4	280,000	0.007	Total number of non-
			parous winged mosquitoes
Y ₅	280,000	0.007	Total number of parous
			winged mosquitoes

Evolve non-infected population through $4,384\ \mbox{days}$ or $12\ \mbox{years}$ and $1\ \mbox{day}.$

BASIC INITIAL CONDITION (BIC) POPULATION

Quantity	Value	Fraction of Total	Description
Y ₁	106,380,800	0.9743	Total number of eggs
Y ₂	949,598	0.0087	Total number of larvae
Y ₃	155,330	0.0014	Total number of pupae
Y4	423,110	0.0039	Total number of non-
			parous winged mosquitoes
Y_5	1,279,318	0.0117	Total number of parous
			winged mosquitoes

* assume sex-ratio of 1/2

ESTABLISHMENT POPULATION THRESHOLD

Apply same sex-ratio to BIC for infected and uninfected mosquitoes

ESTABLISHMENT POPULATION THRESHOLD

- Apply same sex-ratio to BIC for infected and uninfected mosquitoes
- Apply same share proportion for all stages to infected and uninfected population, for instances 0.85 uninfected and 0.15 infected

ESTABLISHMENT POPULATION THRESHOLD

- **1** Apply same sex-ratio to BIC for infected and uninfected mosquitoes
- Apply same share proportion for all stages to infected and uninfected population, for instances 0.85 uninfected and 0.15 infected
- Verify if Wolbachia-infected mosquitoes prevail
- If invaded, reduce share proportion of Wolbachia-infected
- If not invaded, increase share proportion of Wolbachia-infected
- Service 3 to 5 until desired accuracy

TYPICAL RUN - POPULATION THRESHOLD



EXPERIMENTAL PROCEDURE - MOSQUITO RELEASE

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MOSQUITO RELEASES

3 stages: Eggs; or

Larvae; or

Parous winged mosquitoes

Periodicity

Single release at 1st January; or 52 weekly releases for a year

TYPICAL RUN - MOSQUITO RELEASE



RESULTS

POPULATION THRESHOLD

- 0.0089416505 \approx 1% FI
- 0.0089416505 $\approx 1\%$ MI
- 0.49105835 \approx 49% FU
- 0.49105835 ≈ 49% MU

SINGLE RELEASE - PERCENTAGE



SINGLE RELEASE

SINGLE RELEASE - ABSOLUTE NUMBERS



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WEEKLY RELEASE - PERCENTAGE



WEEKLY RELEASE - ABSOLUTE NUMBERS



WEEKLY RELEASE

DENSITY DEPENDENT DEATH EFFECTS



DISCUSSION - Density Dependent Death

- SCALING parameter for Total Population and Release
- Fast effect

DISCUSSION - Minimum Percentage for Invasion

for SINGLE Release 3 % of EGGS sub-population 8 % of PAROUS sub-population 126 % of LARVAE sub-population

DISCUSSION - Minimum Percentage for Invasion

for WEEKLY Releases 0.1 % of EGGS sub-population 0.2 % of PAROUS sub-population 4 % of LARVAE sub-population

DISCUSSION - Practical Implementation

- Different difficulties for each stage
- POPULATION SIZE UNKNOWN
- FIOCRUZ can produce 30,000 parous mosquitoes per week
- for SINGLE Release 50,930 individuals of PAROUS sub-population
- for WEEKLY Releases

454 individuals of PAROUS sub-population

DISCUSSION - in summary

- Invasion is possible
- Feasible with existent means
- Different "efforts"

Thank You!!!!

