Extending Topological Surgery to Natural Processes

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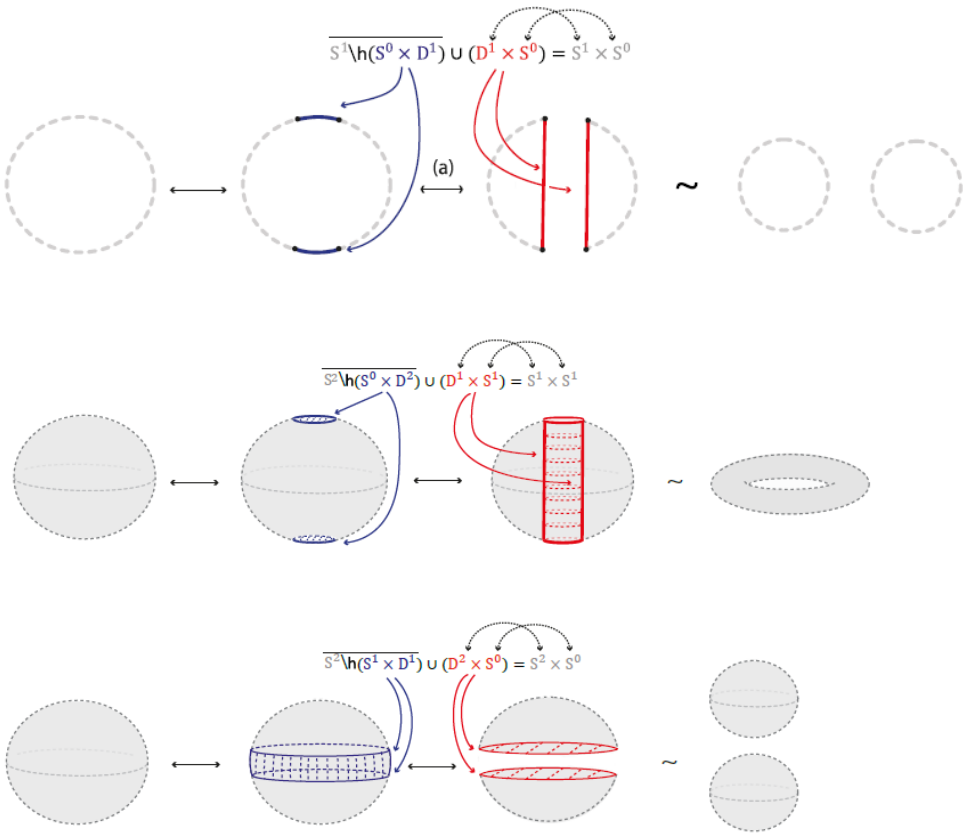
Formal definition of Topological Surgery

n-surgery

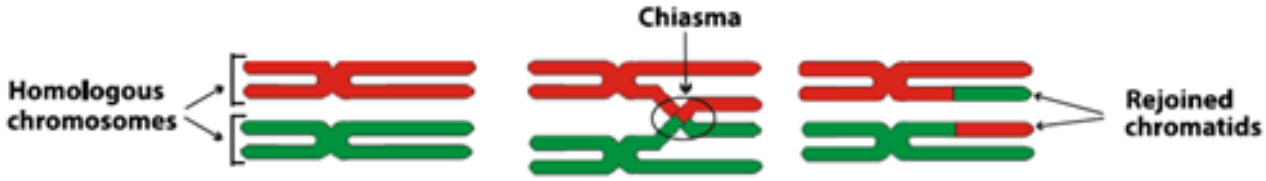
$$M' = \chi(M) = \overline{M \setminus h(S^n \times D^{m-n})} \cup_{h|_{S^n \times S^{m-n-1}}} D^{n+1} \times S^{m-n-1}$$

Dual: m-n-1

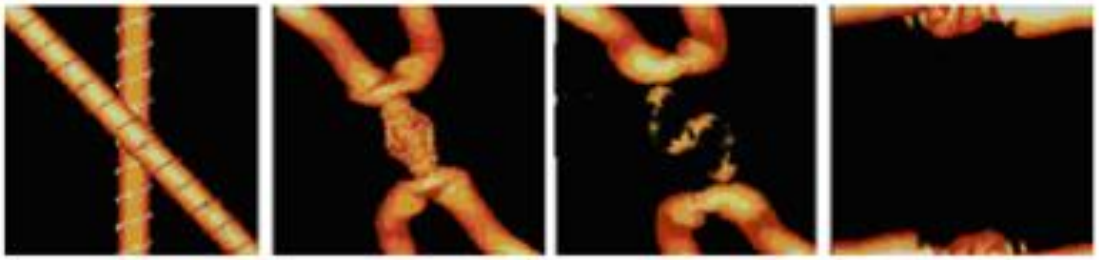
$$M = \chi^{-1}(M') = \overline{M' \setminus g(D^{n+1} \times S^{m-n-1})} \cup_{h^{-1}|_{S^n \times S^{m-n-1}}} S^n \times D^{m-n}$$



1D Surgery in Nature

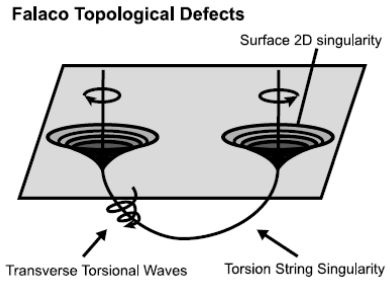


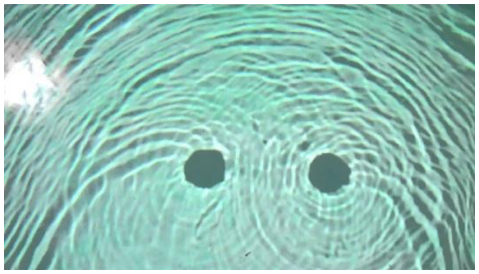
Crossing over of chromosomes during meiosis.



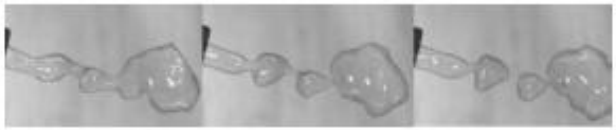
The reconnection of cosmic magnetic lines.

2D Surgery in Nature

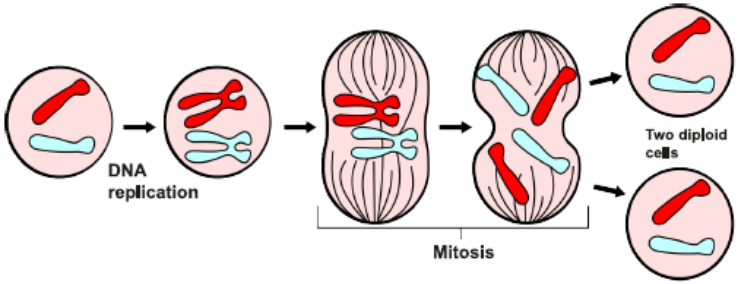


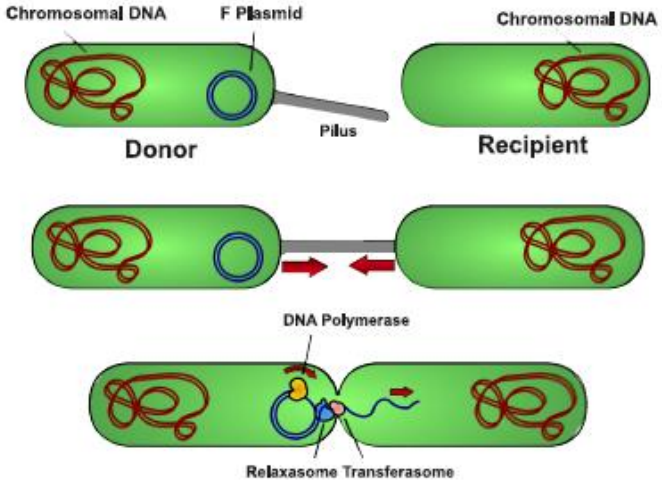


Tornadoes

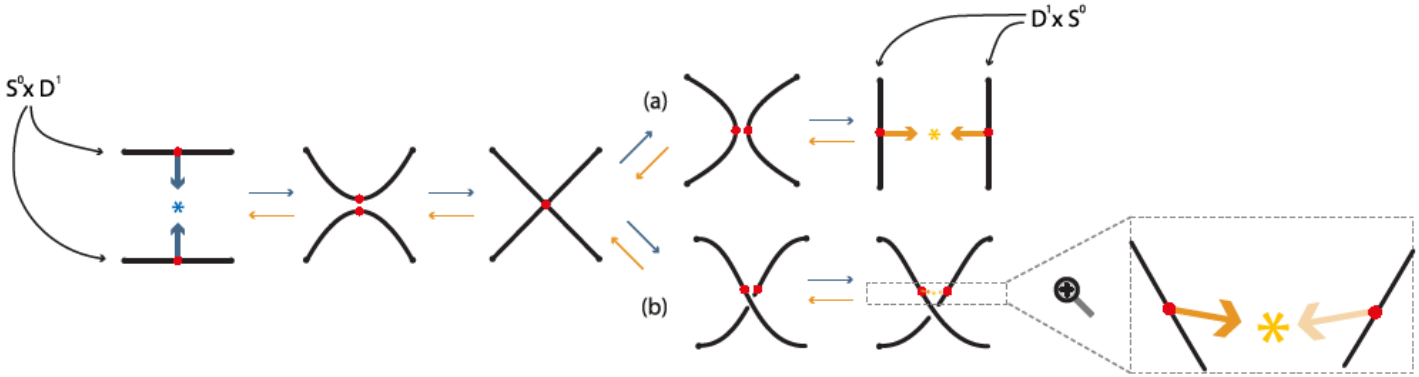


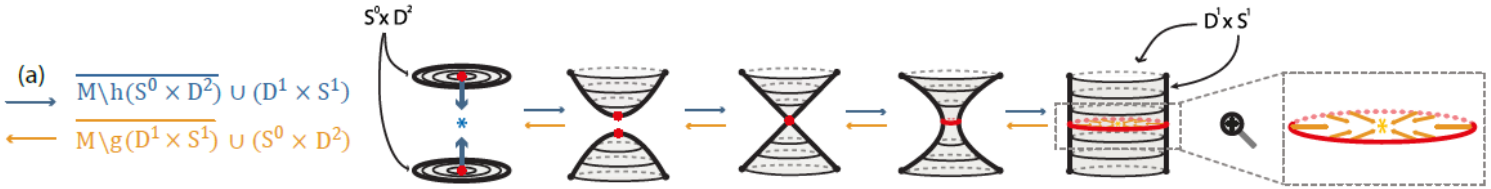
Soap bubble splitting. An example of 2-dimensional 1-surgery.

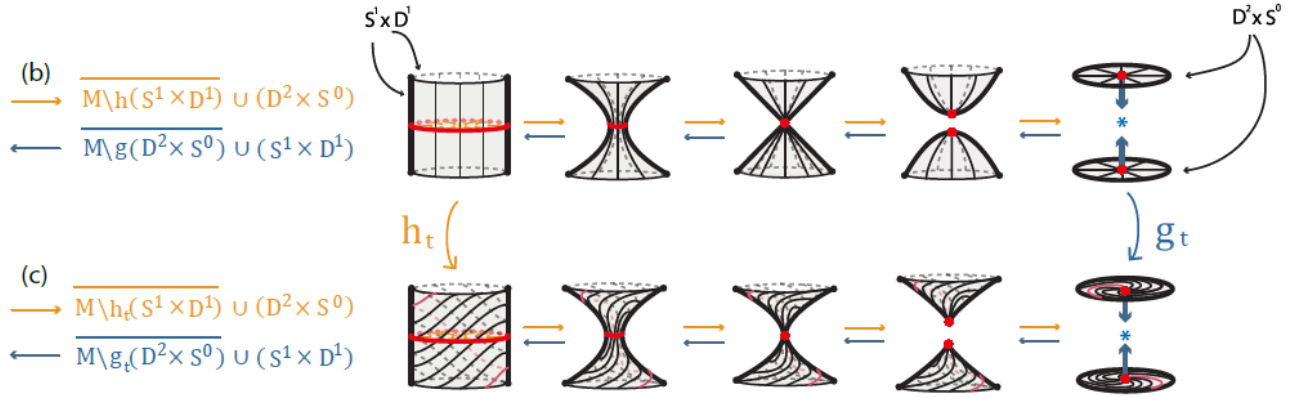




Introducing dynamics



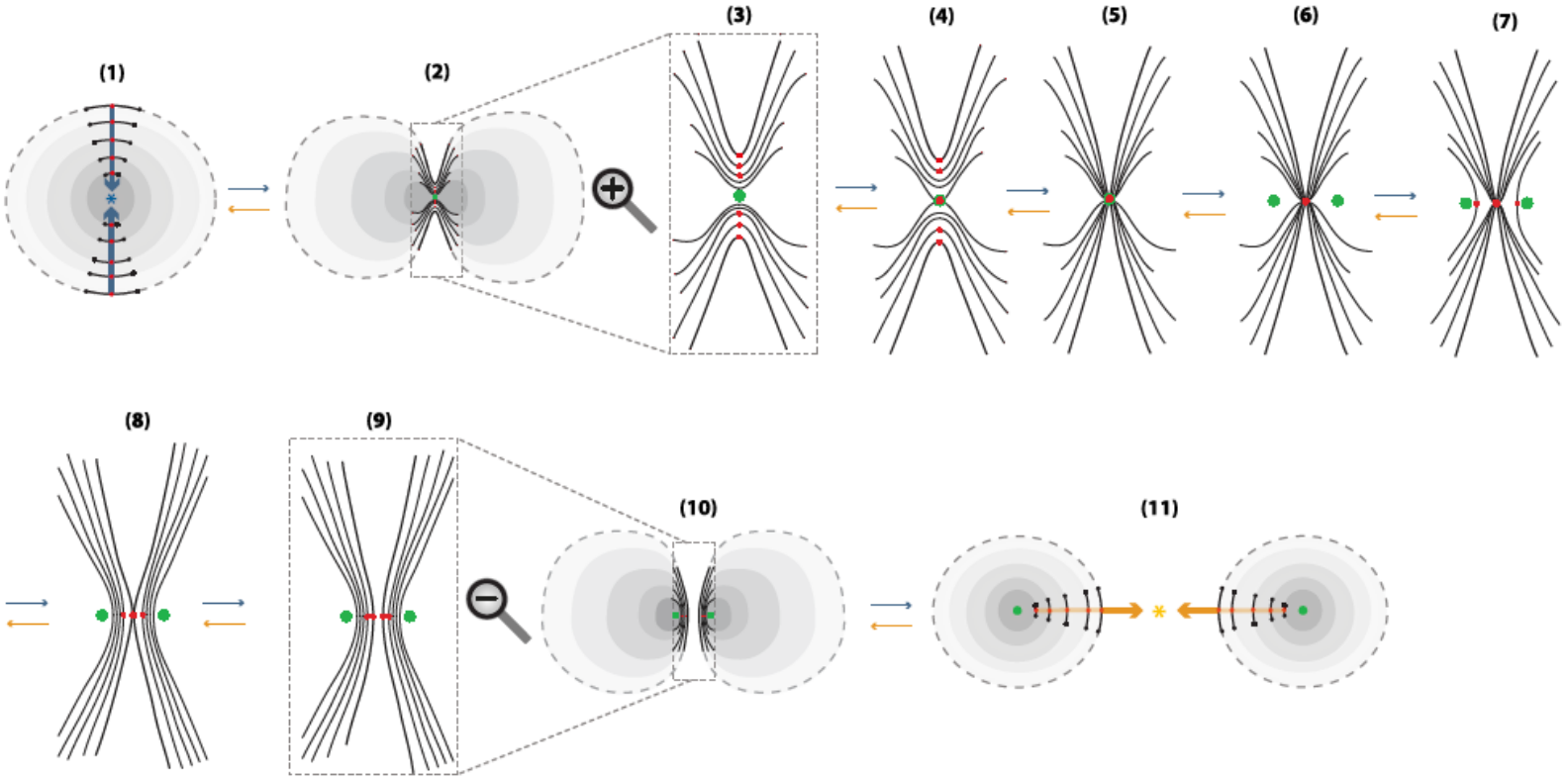




Solid 1-dimensional surgery

$$D^2 = \cup_{0<r\leq 1} S^1_r \cup \{P\} \qquad \chi(D^2) := \cup_{0<r\leq 1} \chi(S^1_r) \cup \chi(P)$$

$$\chi^{-1}(D^2 \times S^0) := \cup_{0<r\leq 1} \chi^{-1}(S^1_r \times S^0) \cup \chi^{-1}(P \times S^0)$$



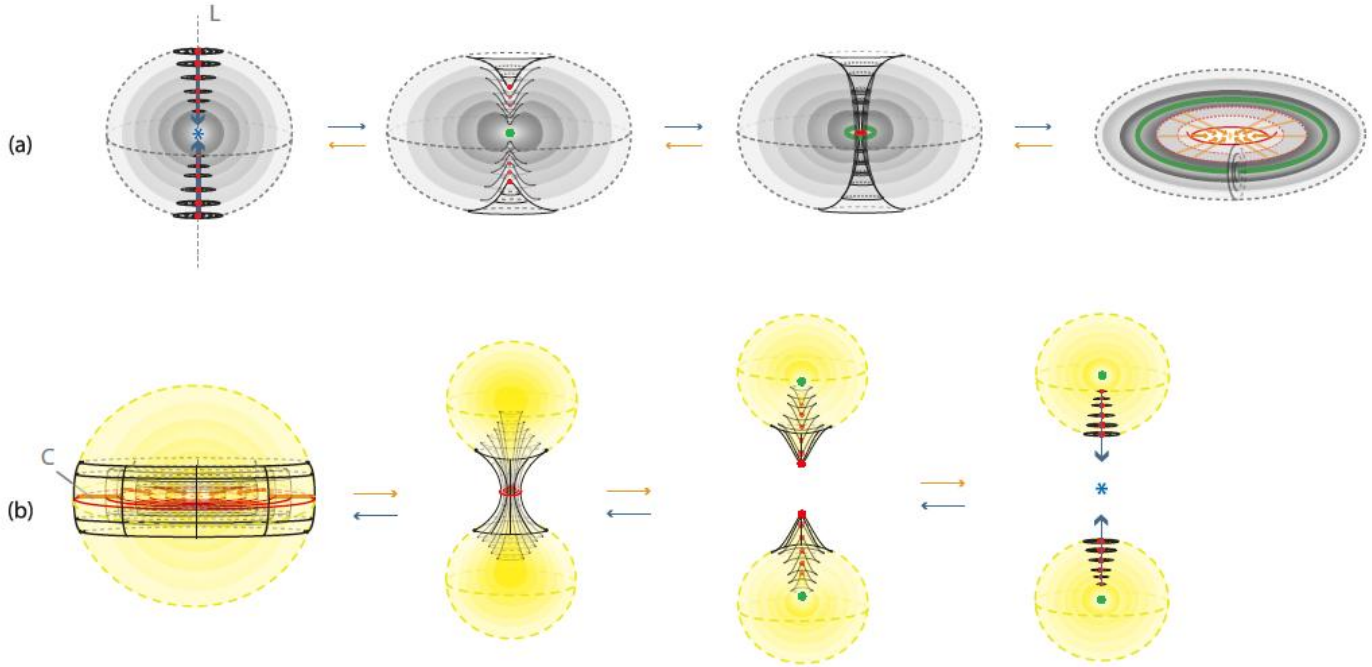
Solid 2-dimensional surgery

$$D^3 = \cup_{0<r\leq 1} S^2_r \cup \{P\}$$

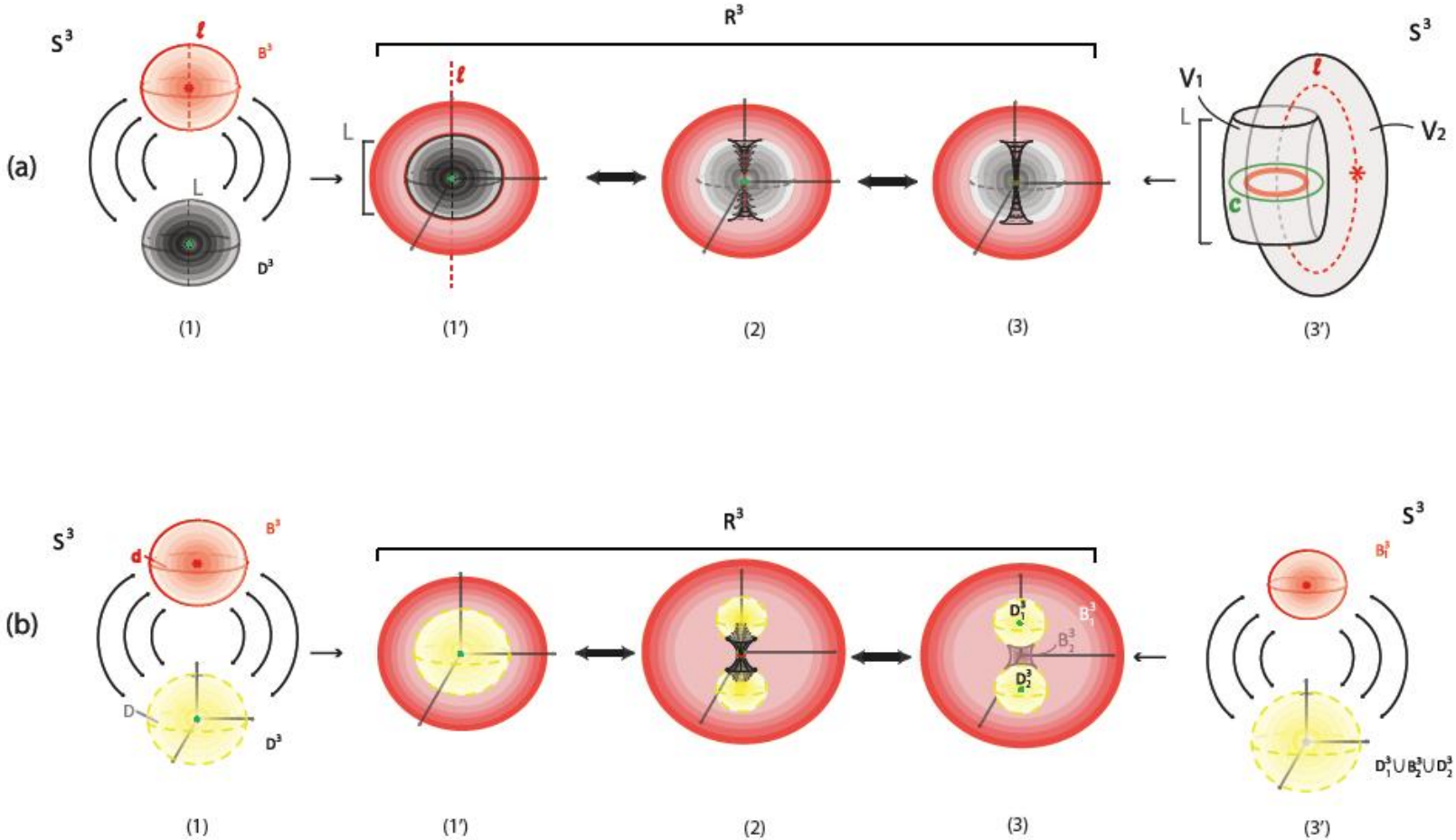
$$\chi(D^3) := \cup_{0<r\leq 1} \chi(S^2_r) \cup \chi(P)$$

$$D^2 \times S^1 = (\cup_{0<r\leq 1} S^1_r \cup \{P\}) \times S^1$$

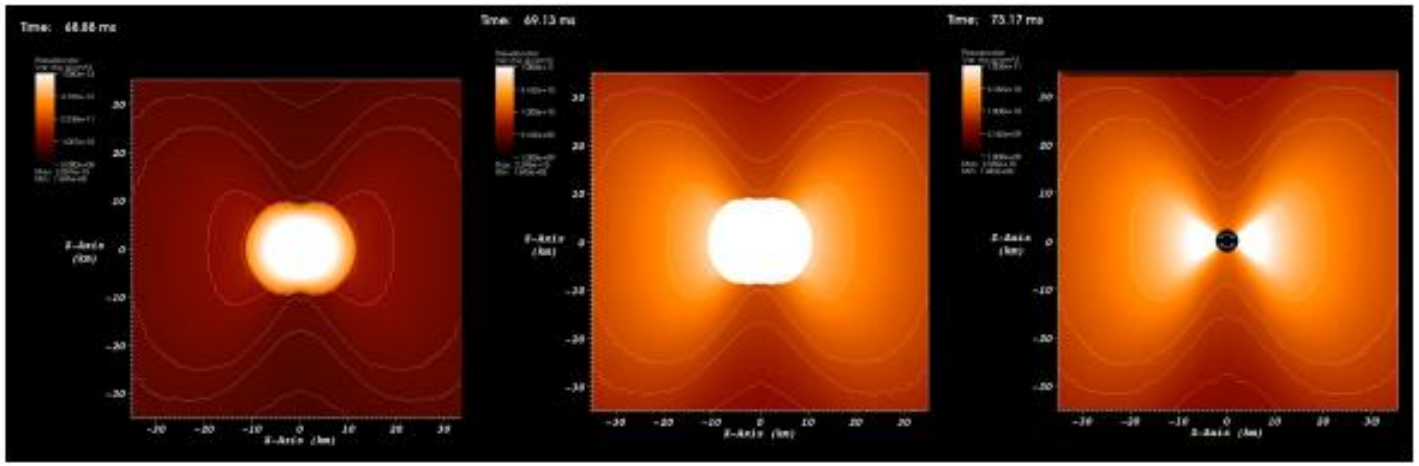
$$\chi_0^{-1}(D^2 \times S^1) := \cup_{0<r\leq 1} \chi_0^{-1}(S^1_r \times S^1) \cup \chi_0^{-1}(P \times S^1)$$



Embedded solid 2D surgeries



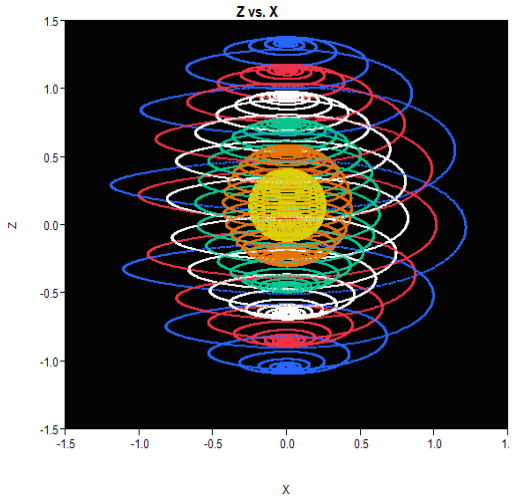
Example of embedded 2D solid 0-surgery

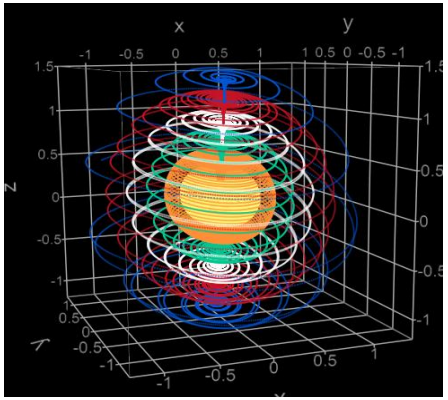


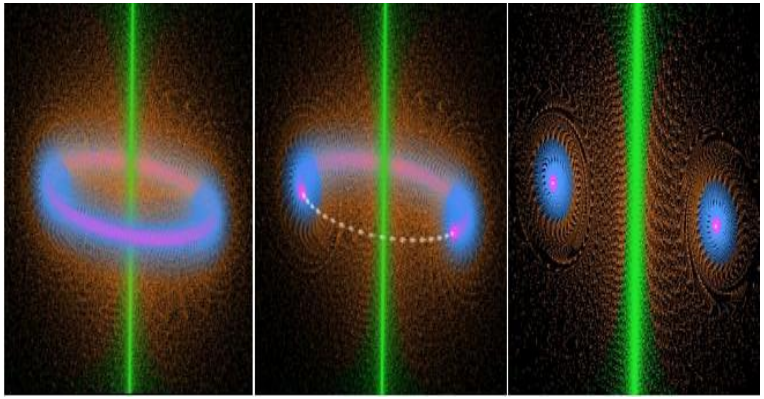
The formation of a black hole.

Work in progress

$$\Sigma: \quad \begin{aligned} \dot{X} &= X - XY + CX^2 - AZX^2 \\ \dot{Y} &= -Y + XY \\ \dot{Z} &= -BZ + AZX^2 \end{aligned}$$







A transformation of the 3D Lotka Volterra system *(images of Nick Samardzija)*

References

Topological Surgery and its Dynamics *(http://arxiv.org/abs/1406.1106v1)*
Dynamical Systems and Topological Surgery *(http://arxiv.org/abs/0812.2367v1)*
S.Lambropoulou website *(http://www.math.ntua.gr/~sofia/papers/extending_topological_surgery_to_natural_processes.pdf)*