Cunha's calculus in its times

João Caramalho Domingues
(Centro de Matemática da Universidade do Minho)
jcd@math.uminho.pt

AMS-EMS-SPM International Meeting
Porto, 10–13 June 2015

Ivor Grattan-Guinness
(1941–2014)

Da Cunha's calculus in its time

by *I. Grattan-Guinness*

Proceedings of *Anastácio da Cunha. O matemático e o poeta*, Lisbon, 8–9 October 1987

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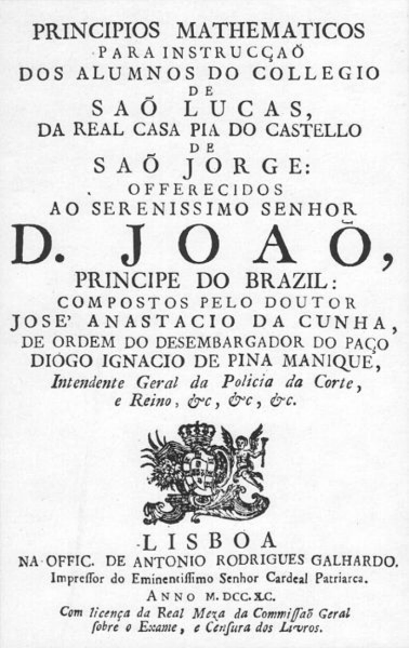

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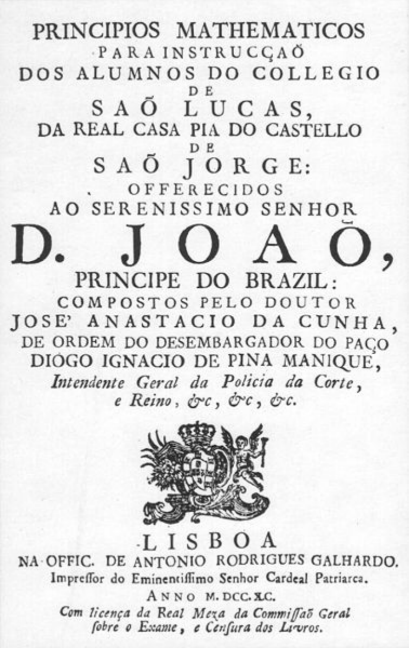
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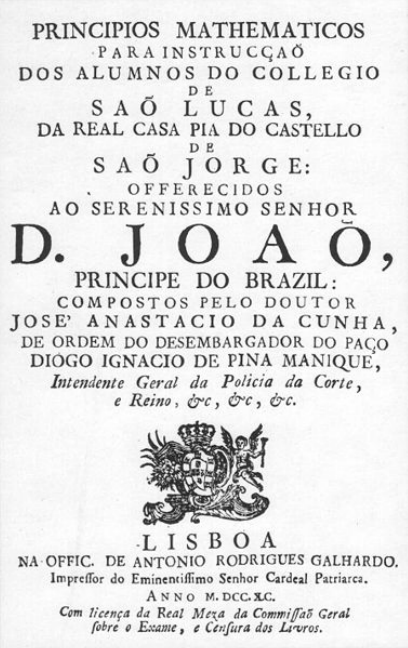
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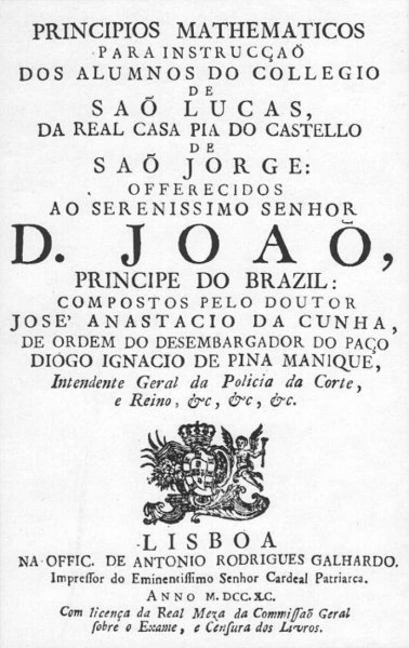
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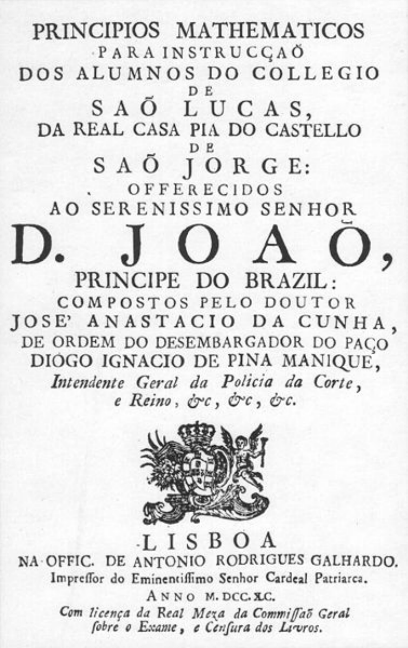

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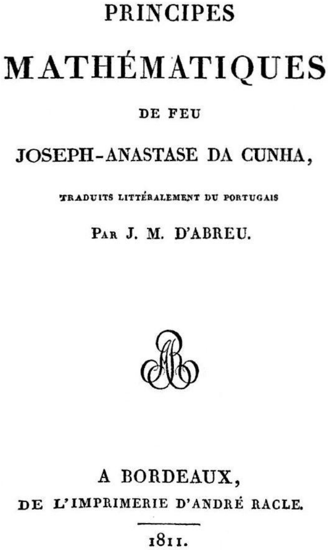
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short references in biographical dictionaries
one elementary geometrical proof used in a British and an American textbooks

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Understatement!

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3. functions and the approach of Euler
   purely analytical; about functions (analytical expressions);
   variables as abstract quantities; little adherence before the 1790s

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- Functions from the outset.

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Grattan-Guinness, 1987: dx and $d\Gamma x$ not properly defined; no restriction on Γx.

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P a magnitude larger than each of them,
and Q any given magnitude;
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Ivor Grattan-Guinness, 1987:

> "Impressive but odd, powerful but cryptic, this book […] 'interesting', but too off-beat to gain the attention he deserved."

The twenty-five year gap

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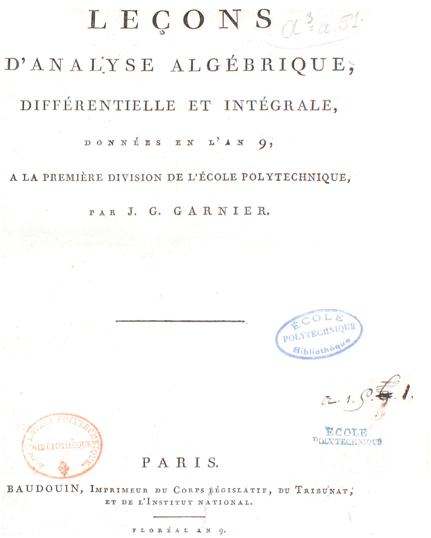
1786–97	reprints and translations of Euler's treatises on the calculus
1794	Pietro Paoli, *Elementi d'Algebra* (modelled on Euler's treatises)
1797	Lagrange, *Théorie des fonctions analytiques*
1797–1800	Lacroix, *Traité du calcul différentiel et du calcul intégral*
1800	Arbogast, *Du calcul des dérivations*
1801, 1806	Lagrange, *Leçons sur le calcul des fonctions*
1805	Poisson, "Démonstration du théorême de Taylor"
1806	Ampère, "Recherches sur quelques points de la théorie des fonctions dérivées […]"
1809	Binet, "Mémoire sur la fonction dérivée […]"



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D'ANALYSE ALGÉBRIQUE,

DIFFÉRENTIELLE ET INTÉGRALE,

DONNÉES EN L'AN 9,

A LA PREMIÈRE DIVISION DE L'ÉCOLE POLYTECHNIQUE,

PAR J. G. GARNIER.

PARIS.

BAUDOUIN, Imprimeur du Corps législatif, du Tribunat, et de l'Institut national.

FLORÉAL AN 9.

École Polytechnique
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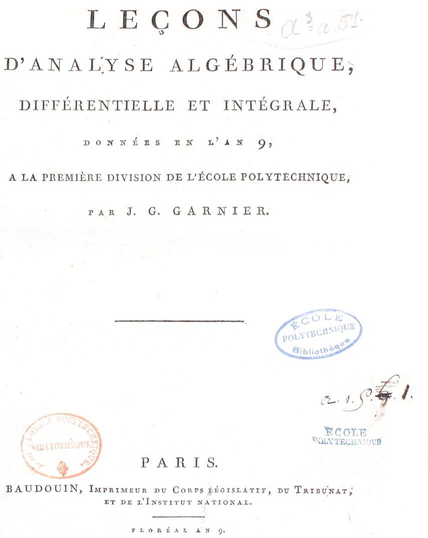
Lagrange, Fourier, Lacroix, …



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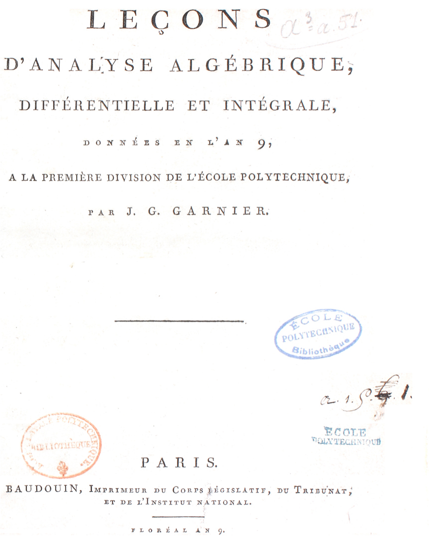

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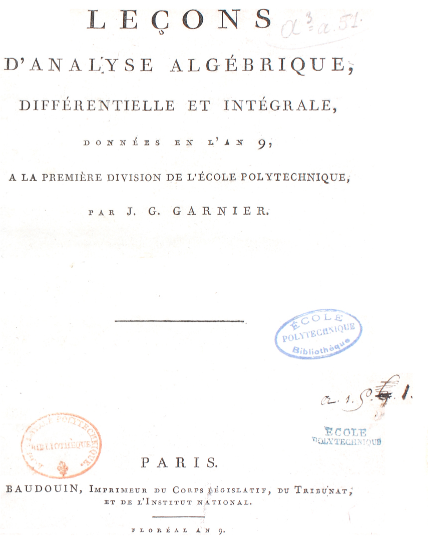

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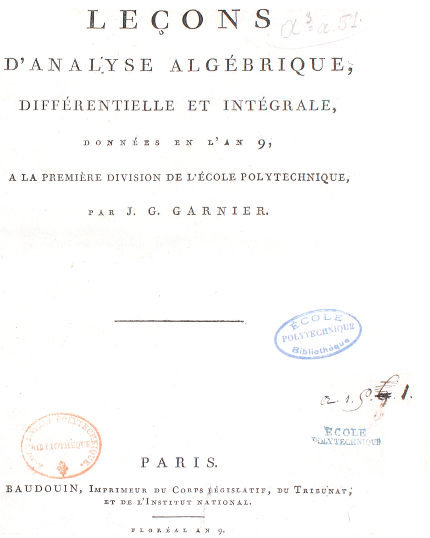
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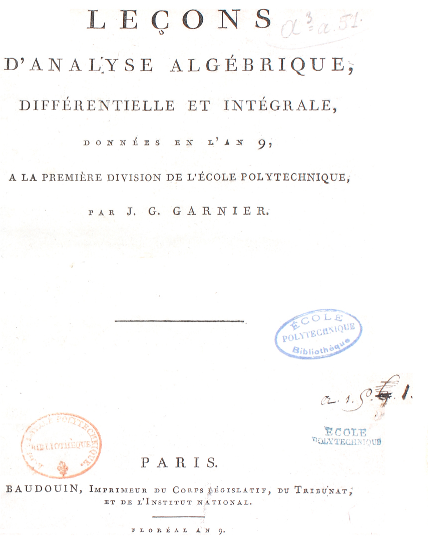
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ET
DE CALCUL INTÉGRAL;

PAR S. F. LACROIX.

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revue et corrigée.

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1806.

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differential coefficient:
limit of $\dfrac{f(x + \mathrm{d}x) - f(x)}{\mathrm{d}x} = p + q\,\mathrm{d}x + \text{etc.}$
which is $p = \dfrac{\mathrm{d}u}{\mathrm{d}x}$

Four reviews of Cunha's *Principes*

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- *Edinburgh Review*, 1812, anonymous (John Playfair)
  "the definition of a fluxion is very difficult to be understood [...] How much better it would have been, to call the fluxion of any function *the first term of the increment of that function*, which, after all, is the idea meant to be conveyed."

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  “To the word *infinitesimal* he applies not the idea of an infinitely small quantity, but rather of a variable that may become smaller than any given magnitude: being different only in name from the indeterminate increments of the variables according to the new methods.

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$df(x)$ such that $\dfrac{df(x)}{dx}$ constant and $\dfrac{f(x+dx)-f(x)}{dx} - \dfrac{df(x)}{dx}$ infinitesimal if dx were infinitesimal	$f(x+\mathrm{d}x) - f(x) = p\,\mathrm{d}x + q\,\mathrm{d}x^2 + \text{etc.}$

Vincenzo Brunacci, *Memoria premiata dall'Accademia di Padova*, 1810
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Lagrangian calculus of funcions is a branch of ordinary algebra: no new operations, no new principles, quantities regarded in the same way.

While Leibnizian infinitesimal calculus needs infinitesimal quantities;

and the method of limits considers quantities at the very momento when they cease to be quantities.

Did Cunha anticipate the modern definition of differential?

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Did Cunha anticipate the definition of differential of the Lagrangian tradition?