Nonstandard Intuitionistic Interpretations

We present a notion of realizability and a functional interpretation in the context of intuitionistic logic, both incorporating nonstandard principles.

In a recent paper Ferreira and Gaspar [4] showed how the bounded functional interpretation of [5] can be recast without intensional notions by going to a wider nonstandard setting. This was carried out in the classical setting. The bounded functional interpretation relies on the Howard/Bezem notion of strong majorizability introduced in [6] and [3] (see also [8]). The functional interpretation that we present corresponds to the intuitionistic counterpart of the interpretation given in [4]. It has some similarities with [1] but it replaces finiteness conditions by majorizability conditions.

Nonstandard methods are often regarded as nonconstructive. Our interpretations intend to seek for constructive aspects in nonstandard methods (in the spirit of, say, [1] and [2]).

References

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