Abstract

A three-parameter family $B = B(a, b, c)$ of weighted Hankel matrices is introduced with the entries

$$B_{j,k} = \frac{\Gamma(j+k+a)}{\Gamma(j+k+b+c)} \sqrt{\frac{\Gamma(j+b)\Gamma(j+c)\Gamma(k+b)\Gamma(k+c)}{\Gamma(j+a)\, j!\, \Gamma(k+a)\, k!}}\,,$$

$j, k \in \mathbb{Z}_+$, assuming that a, b, c are positive and $a < b+c$, $b < a+c$, $c < a+b$. The famous Hilbert matrix given by $B_{j,k} = 1/(j+k+\theta)$ is included as a particular case for $a = b = \theta$, $c = 0$. The direct sum

$$B(a, b, c) \oplus B(a+1, b+1, c)$$

is shown to commute with a discrete analogue of the dilatation operator. It follows that there exists a three-parameter family $T(a, b, c)$ of real symmetric Jacobi matrices such that $T(a, b, c)$ commutes with $B(a, b, c)$. The members of the orthogonal polynomial sequence associated with $T(a, b, c)$ are the continuous dual Hahn polynomials. Since the corresponding measure of orthogonality is known explicitly, a unitary mapping U diagonalizing $T(a, b, c)$ can be constructed explicitly. The spectrum of $T(a, b, c)$ is simple and therefore U diagonalizes $B(a, b, c)$ as well. It turns out that the spectrum of B as an operator on $\ell^2(\mathbb{Z}_+)$ is purely absolutely continuous,

$$\operatorname{spec} B(a, b, c) = [0, M(a, b, c)], \text{with } M(a, b, c) = \frac{\Gamma\big((b+c-a)/2\big)^2}{\Gamma(b+c-a)}\,.$$

If the assumption $c < a+b$ is relaxed while the remaining inequalities on a, b, c are all supposed to be valid, the spectrum contains also a finite discrete part lying above the upper threshold of the continuous spectrum.

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