Free idempotent generated semigroups and the endomorphism monoid of a free G-act

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The study of the free idempotent generated semigroup $\mathrm{IG}(E)$ over a biordered set E began with the seminal work of Nambooripad [7] in the 1970s and has seen a recent revival with a number of new approaches, both geometric and combinatorial. A particular focus, which this talk explores, has been on the maximal subgroups of $\mathrm{IG}(E)$. A long-standing conjecture that all such subgroups were free was shown to be false, first by a counterexample of Brittenham, Margolis and Meakin [1], and later by a proof by Gray and Ruškuc [5] that *any* group arises in this way.

It follows from a result of Easdown [3] that we may assume that E is the biordered set of idempotents of an idempotent generated semigroup S. Given such a 'natural' S, what is the relation between the maximal subgroups of $\mathrm{IG}(E)$ and those of S? In particular, when is H_e (the maximal subgroup of S with identity $e \in E$) isomorphic to $H_{\overline{e}}$ (the corresponding subgroup of $\mathrm{IG}(E)$)?

Here we consider $\mathrm{IG}(E)$ in the case E is the biordered set of a wreath product $G \wr \mathcal{T}_n$, where G is a group and $\mathcal{T}_n$ is the full transformation monoid on n elements. This wreath product is isomorphic to the endomorphism monoid of the free G-act $\mathrm{End}\, F_n(G)$ on n generators, and this provides us with a convenient approach.

We say that the *rank* of an element of $\mathrm{End}\, F_n(G)$ is the minimal number of (free) generators in its image. Let $\varepsilon = \varepsilon^2 \in \mathrm{End}\, F_n(G)$. For rather straightforward reasons it is known that if $\mathrm{rank}\, \epsilon = n-1$ (respectively, n), then the maximal subgroup of $\mathrm{IG}(E)$ containing ε is free (respectively, trivial). Taking $r = 1$ a relatively elementary argument gives $H_{\overline{\varepsilon}} \cong H_\varepsilon \cong G$, providing an alternative approach to the result of Gray and Ruškuc [4]. For higher ranks, we need to build on the sophisticated techniques developed in [5]. We show that if $\mathrm{rank}\, \epsilon = r$ where $1 \leq r \leq n-2$, then $H_{\overline{\varepsilon}}$ is isomorphic to H_ε and hence to $G \wr \mathcal{S}_r$, where $\mathcal{S}_r$

is the symmetric group on r elements. Taking G to be trivial, we obtain an alternative proof of a result of Gray and Ruškuc [6] for the biordered set of idempotents of $\mathcal{T}_n$.

This is joint work with Yang Dandan and Igor Dolinka [2].

References

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