Lax pairs and Riemann-Hilbert problems for conjugate conductivity equations

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We use the unified transform method to derive explicit integral expressions for the solutions of the conductivity equations

$$\Delta u + \frac{p}{x}\partial_x u = 0, \qquad p \in \mathbb{Z},$$

in a bounded domain of the right half-plane. These equations are the topic of the so-called axially symmetric potential theory, which was initiated by A. Weinstein in the 50's.

The present method is based on the use of Lax pairs and the study of a specific Riemann-Hilbert problem, depending on the parity of p. The existence of a Lax pair can be expressed as the closeness of a differential form. We show how this global relation allows for an explicit correspondance between Dirichlet and Neumann boundary values. This also involves a result by Karlin and Szegö on wronskians of orthogonal polynomials.

This is a joint work with Slah Chaabi and Stéphane Rigat.

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