Genera of non-algebraic leaves of polynomial foliations of $\mathbb{C}^2$

We consider the space $\mathcal{A}_n$ of polynomial foliations of $\mathbb{C}P^2$ given by

$$\begin{cases} \dot{x} = p(x, y) \\ \dot{y} = q(x, y) \end{cases}$$

in a fixed affine chart, $\deg p = \deg q = n$. It is well-known that each leaf of a generic foliation from $\mathcal{A}_n$ is dense in $\mathbb{C}P^2$.

We prove that in a dense subset of $\mathcal{A}_n$ any foliation has a (non-algebraic) leaf with at least $\frac{(n+1)(n+2)}{2} - 4$ handles. This result is motivated by an earlier result of D.Volk: in a dense subset of $\mathcal{A}_n$, any foliation has a separatrix connection.

We also prove that all non-algebraic leaves of any foliation given by

$$\begin{cases} \dot{x} = p(x^2, y) \\ \dot{y} = xq(x^2, y) \end{cases}$$

have infinite genus.