

Magnetic complex structures

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Let (M, g) be a real-analytic Riemannian manifold, equipped with a closed 2-form β , interpreted as a magnetic field on M . Let Φ_t^β be the Hamiltonian flow on $T^*(M)$ describing the motion of a charged particle on M in the presence of the magnetic field β . Following an idea of T. Thiemann, we construct complex structures on $T^*(M)$ by pushing forward the vertical polarization by the “imaginary-time flow” $\Phi_{i\tau}^\beta$, $\tau > 0$. When $\beta = 0$, the complex structures coincide with the adapted complex structure of Lempert–Szőke and Guillemin–Stenzel. We give several different ways of understanding the imaginary-time flow, we construct a local Kähler potential, and we give explicit examples on the plane and the sphere. My talk also describes work with J. Mitchell on the quantum side of this program.