Supersymmetry decomposes the virtual bundle that underlies the elliptic genus

Geometrical and Enumerative Structures in Supersymmtry,
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The elliptic genus under extended supersymmetry

[Creutzig/W15], *work in progress*

[W15] *K3 en route from geometry to conformal field theory*; to appear in: Proceedings of the 2013 Summer School "Geometric, Algebraic and Topological Methods for Quantum Field Theory", Villa de Leyva, Colombia; arXiv:1503.08426 [math.DG]

[W14] *Snapshots of conformal field theory*; in "Mathematical Aspects of Quantum Field Theories", Mathematical Physics Studies, Springer 2015, pp. 89-129; arXiv:1404.3108 [hep-th]

[Taormina/W13] *A twist in the M_{24} moonshine story*, to appear in Confluentes Mathematici; arXiv:1303.3221 [hep-th]

[Taormina/W12] *Symmetry-surfing the moduli space of Kummer K3s*, to appear in: Proceedings of String-Math 2012; arXiv:1303.2931 [hep-th]

[Taormina/W11] *The overarching finite symmetry group of Kummer surfaces in the Mathieu group M_{24}*, JHEP **1308**:152 (2013); arXiv:1107.3834 [hep-th]

1. The elliptic genus of Calabi-Yau manifolds...

Let M denote a compact Calabi-Yau D-fold, $T := T^{1,0}M$,

$$\mathbb{E}_{q,-y} := y^{-\frac{D}{2}}\Lambda_{-y}T^* \otimes \bigotimes_{n=1}^{\infty} \left[\Lambda_{-yq^n}T^* \otimes \Lambda_{-y^{-1}q^n}T \otimes S_{q^n}T^* \otimes S_{q^n}T\right],$$

where for any bundle $E\to M$, $\Lambda_x E := \bigoplus_{m=0}^{\infty} x^m \Lambda^m E$, $S_x E := \bigoplus_{m=0}^{\infty} x^m S^m E$

Definition

With $q := e^{2\pi i\tau}$, $y := e^{2\pi iz}$ for $\tau, z \in \mathbb{C}$, $\mathrm{Im}(\tau) > 0$,

$$\mathcal{E}_M(\tau, z) := \chi(\mathbb{E}_{q,-y}) = \int_M \mathrm{Td}(M)\,\mathrm{ch}(\mathbb{E}_{q,-y})$$

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For M: a K3 surface,

that is, a compact Calabi-Yau 2-fold with $h^{1,0}(M) = 0$,

$$\mathcal{E}_{K3}(\tau, z) = 8\left(\frac{\vartheta_2(\tau,z)}{\vartheta_2(\tau,0)}\right)^2 + 8\left(\frac{\vartheta_3(\tau,z)}{\vartheta_3(\tau,0)}\right)^2 + 8\left(\frac{\vartheta_4(\tau,z)}{\vartheta_4(\tau,0)}\right)^2.$$

1. . . . and for superconformal field theories

CFT elliptic genus

of an $N=(2,2)$ SCFT at central charges $(c,\overline{c})$ with space-time SUSY and integral $U(1)$ charges:

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$\mathcal{H}_R$: Ramond sector,

$J_0, L_0, \overline{L}_0$: zero modes of the $U(1)$-current and Virasoro fields in the SCA

For every $N=(2,2)$ SCFT at central charges $c=\overline{c}=6$ with space-time SUSY and integral $U(1)$ charges, the CFT elliptic genus $\mathcal{E}_{CFT}(\tau,z)$ either vanishes, or it agrees with $\mathcal{E}_{K3}(\tau,z)$. The theory has $N=(4,4)$ SUSY.

Examples:

toroidal SCFTs, their orbifolds, Gepner models at $c=\overline{c}=6$

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A K3 THEORY is an $N=(4,4)$ SCFT at $c=\overline{c}=6$ with space-time SUSY, integral $U(1)$ charges and CFT elliptic genus $\mathcal{E}_{K3}(\tau,z)$.

2. Decomposing under $N = (4,4)$ supersymmetry

Assume: $D = 2$, i.e. $c = \overline{c} = 6$ and $N = (4,4)$ supersymmetry.

3 types of $N = 4$ irreps $\mathcal{H}_\bullet$ with $\chi_\bullet(\tau, z) = \mathrm{sTr}_{\mathcal{H}_\bullet}\left(y^{J_0} q^{L_0 - 1/4}\right)$:
VACUUM $\mathcal{H}_0$, MASSLESS MATTER $\mathcal{H}_{\mathrm{m.m.}}$, MASSIVE MATTER $\mathcal{H}_{h>0}$.

$$\mathcal{E}_{K3}(\tau, z) = -2\chi_0(\tau, z) + 20\chi_{m.m.}(\tau, z) + \sum_{n=1}^{\infty} A_n \chi_n(\tau, z)$$

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Conjecture [W13]
Let M=K3. There are polynomials p_n for every $n \in \mathbb{N}$, such that

$$\mathbb{E}_{q,-y} = -\mathcal{O}_{\mathrm{K3}}\chi_0(\tau, z) \oplus (-T)\chi_{\mathrm{m.m.}}(\tau, z) \oplus \bigoplus_{n=1}^{\infty} p_n(T)\chi_n(\tau, z),$$

where $A_n = \int_{\mathrm{K3}} \mathrm{Td}(\mathrm{K3}) p_n(T) = \chi(p_n(T))$ for all $n \in \mathbb{N}$.

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proof: **[Creutzig/W14]**

Decomposing the virtual bundle: Ideas of proof

Observation:

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$$\overset{\text{holonomy distribution}}{\Longrightarrow} \qquad \mathbb{E}_{q,-y} \cong \bigoplus_{n=1}^{\infty} W_n \otimes M_n,$$

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Conclusion (using **[Malikov/Schechtman/Vaintrob99]**):
$W_n = S^{n-1}(T)$ is invariant under the $N = 4$ SUSY action,

$$\mathbb{E}_{q,-y} \cong \bigoplus_{n=1}^{\infty} W_n \otimes M_n = \bigoplus_{n=1}^{\infty} S^{n-1}(T) \kappa_n(\tau, z),$$

where each $\kappa_n(\tau, z)$ decomposes into $N = 4$ characters.

3. A conjecture of Eguchi, Ooguri and Tachikawa (2010)

$$\mathcal{E}_{K3}(\tau,z) = -2\chi_0(\tau,z) + 20\chi_{m.m.}(\tau,z) + \sum_{n=1}^{\infty} A_n\chi_n(\tau,z),\ \chi_\bullet(\tau,z) = \mathrm{sTr}_{\mathcal{H}_\bullet}\left(y^{J_0}q^{L_0-1/4}\right)$$

Theorem [Gannon12] using results of **Cheng, Duncan, Gaberdiel, Hohenegger, Persson, Ronellenfitsch, Volpato**

There exists a representation $\mathcal{R}_n$ of M_{24} for every $n\in\mathbb{N}$, s.th.

$$\mathcal{R}_{\mathrm{Gan.}} := (-2)\mathcal{H}_0 \ \oplus\ 20\,\mathcal{H}_{\mathrm{m.m.}} \ \oplus\ \bigoplus_{n=1}^{\infty}\mathcal{R}_n\otimes\mathcal{H}_n$$

with $\mathcal{E}_{K3}^{(g)}(\tau,z)=\mathrm{sTr}_{\mathcal{R}_{\mathrm{Gan.}}}\left(g\,y^{J_0}q^{L_0-1/4}\right)$, $g\in M_{24}$, "twisted genera".

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Theorem [Mukai88]

If G is a symmetry group of a K3 surface M,

that is, G fixes the two-forms that define the hyperkähler structure of M,

then G is isomorphic to a subgroup of the Mathieu group M_{24},

and $|G| \leq 960 \ll 244.823.040 = |M_{24}|$.

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Solving Mathieu Moonshine by Symmetry Surfing?

Conjecture **[Taormina/W10-13]**

In every geometric interpretation, we have $\mathcal{H}_R \twoheadrightarrow \mathcal{H}_{R,gen}$, where

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as a representation of the geometric symmetry group $G \subset M_{24}$;

$\mathcal{R}_{\mathrm{Gan.}}$ collects symmetries from distinct pts. of the moduli space.

We call this procedure SYMMETRY SURFING.

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Results [Taormina/W11&12&13]

Restricting to the geometric $\mathbb{Z}_2$-orbifold CFTs on K3:

- The joint action of all geometric symmetry groups yields the maximal subgroup $(\mathbb{Z}_2)^4 \rtimes A_8 \subset M_{24}$.

  Note: $(\mathbb{Z}_2)^4 \rtimes A_8$ is not a subgroup of M_{23}.
- $\mathcal{R}_1$ arises as a space of common states with an action of $(\mathbb{Z}_2)^4 \rtimes A_8$ induced from the $\mathbf{45} \oplus \overline{\mathbf{45}}$ of M_{24}.

  Note: There is a twist in this action.

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Thank you
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