

Title: On injectivity of locales and spaces

Two well-known important facts are the characterization of the continuous lattices as the spaces injective with respect to embeddings in the category \mathbf{Top}_0 of T_0 topological spaces (D. Scott, 1972), and the characterization of the stably locally compact locales as the locales injective with respect to flat embeddings in the category \mathbf{Loc} of locales (P. T. Johnstone, 1981). We show that in \mathbf{Loc} flat embeddings are precisely those morphisms with respect to which a certain finite subcategory is Kan-injective. As a consequence, the category of stably locally compact locales (with convenient morphisms) is the Kan-injective hull of a finite subcategory of \mathbf{Loc} . More generally, we characterize n -flat embeddings in \mathbf{Loc} for each cardinal n by means of Kan-injectivity of finite subcategories and, as a corollary, we obtain analogous characterizations of the n -flat embeddings in \mathbf{Top}_0 . Furthermore, several well-known subcategories of \mathbf{Loc} and \mathbf{Top}_0 are Kan-injective hulls of finite subcategories, and \mathbf{Loc} is the Kan-injective hull of a subcategory of spatial locales. This is joint work with Margarida Carvalho.